Brines Acoustics

Quarter Wave Resonators

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The whole purpose of speaker cabinets is to do something with the sound energy coming off of the back of the driver. A sealed box captures the back wave and prevents it from adding or subtracting from the front wave. A ported box (bass reflex -- BR) resonates at a specific frequency and that energy is added to the front wave. Horns increase the efficiency of the back wave by changing it's acoustic impedance. Quarter wave resonators resonate at a specific frequency as do ported boxes, but the physics are different.

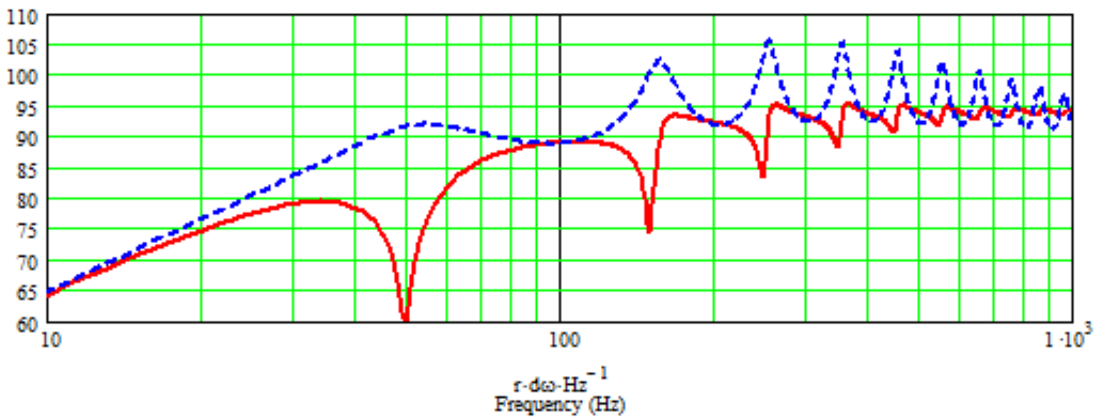
Bass reflex boxes work on the Helmholtz resonator principle. The entire volume of air inside the bass reflex box oscillates as a pressure wave. There are no standing waves. A quarter wave resonator has a series of standing waves between the top of the box and the port. The type of resonator will depend on the shape of the box. A short, fat box will be a Helmholtz resonator and a tall, thin box will be a quarter wave resonator. I will not go into details of exactly what "tall, thin" is here.

What follows here is a simulation of several different pipe geometries. These are simulations using Martin King's MathCAD worksheets. The driver used for these simulations is the Fostex FE167E. This driver works in all of the geometries because it has moderate Q_{ts} and V_{as} values of 0.34 and 32 liters respectively. The Fostex FE167E is not necessarily optimum for any given simulation, and no effort has been made to optimize the simulations other than to tune the system to 50Hz and move the driver so as to cancel the first or second overtone. Where possible, the cabinet volume is made close to V_{as}.

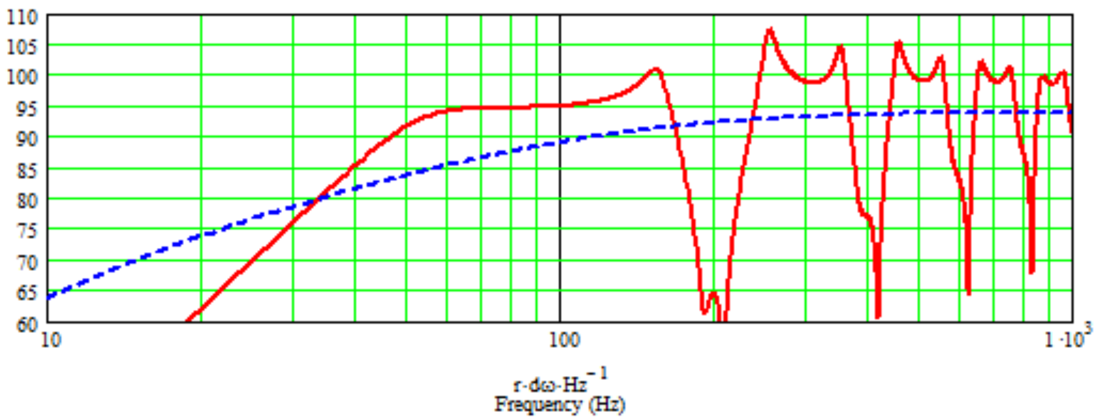
Classic TL

The pipe is 62½" long, S_0 and S_L are set to $3*S_d$. The driver is at the closed end of the pipe.

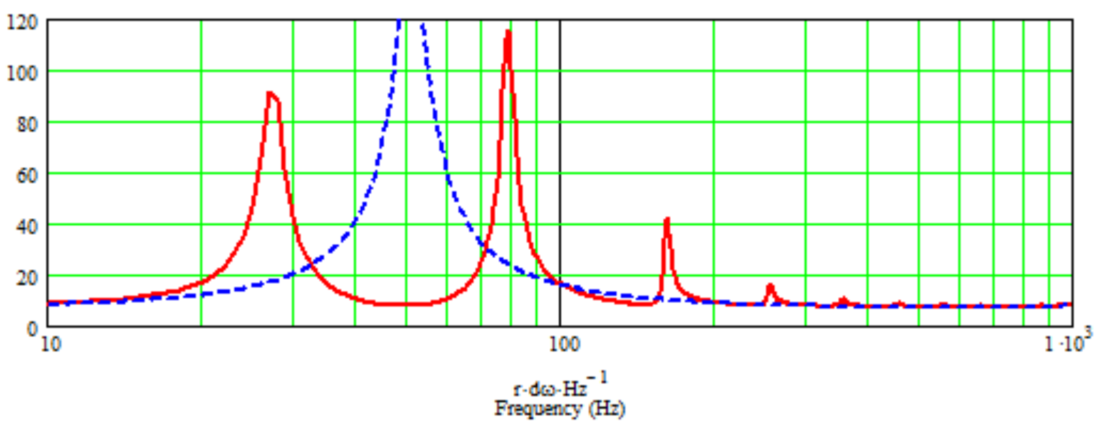
The classic TL is a straight pipe with the driver mounted at one end. The pipe length is cut to tune the system to the driver f_s and is acoustically ¼ wave length. For this simulation, the driver is the Fostex FE167E.



Driver(red) and Port(blue) Output



Combined Driver and Port Output(red) and Infinite Baffle Output(blue)



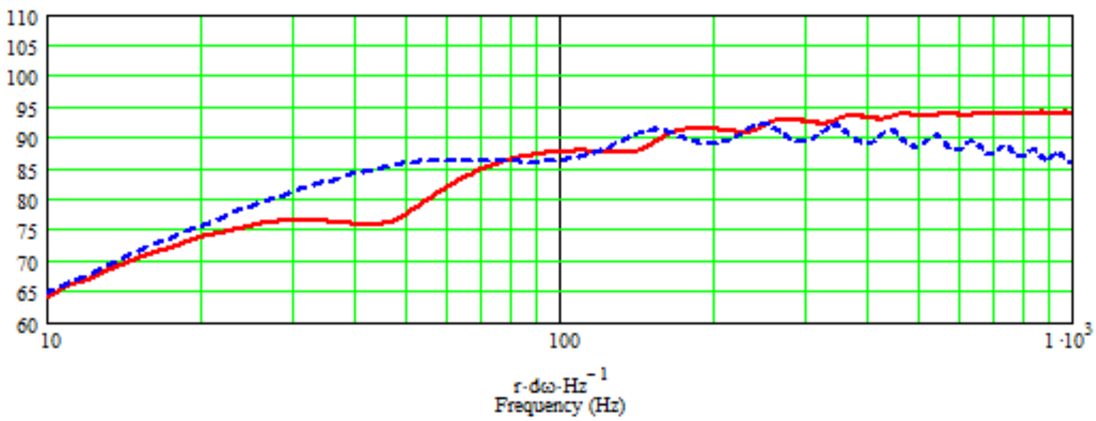
TL Impedance(red) and Infinite Baffle Impedance.(blue)

The driver response of a un-tapered TL follows the response of the driver mounted on an infinite baffle. The difference is that energy is taken from the driver at pipe resonance points. Therefore, the dips in the driver frequency response correspond exactly to the peaks in the port frequency response. Since this is a closed pipe, only odd harmonics are present. At each resonate frequency, the pipe suppresses the output of the driver. In between the resonant peaks, the pipe supplies horn loading to the system, i.e., the port output does not return to zero between harmonics. The summation depends upon the relative phase of the pipe and driver. The pipe is alternately in and out of phase with the driver. This caused the lumpy bass (technically, a comb filter) associated with poorly designed TL speakers.

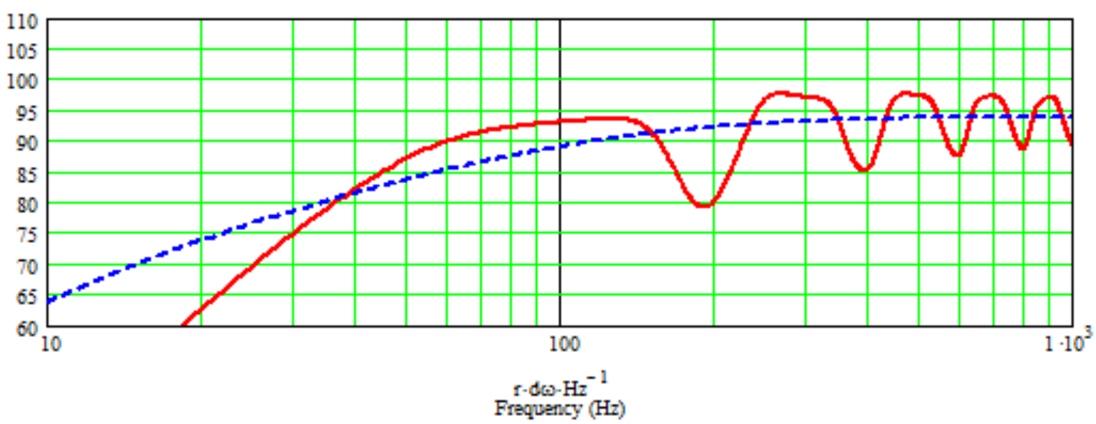
These are the characteristics of an unstuffed classic TL:

- Sharp 24dB/octave cut-off
- A double humped impedance curve
- Strong combined output at the bottom end
- The driver and port are out of phase at every other harmonic creating huge suck-outs
- The flat to cut-off response will be boomy in-room

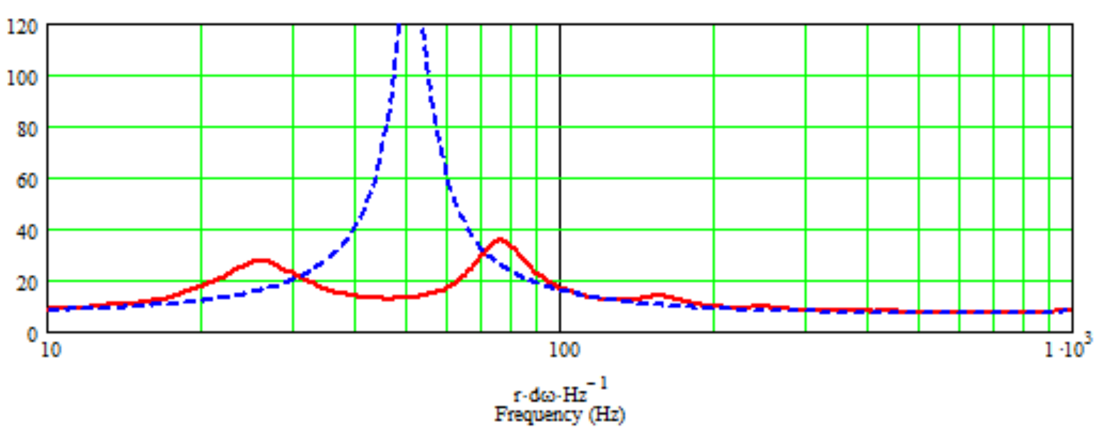
This type of response might be suitable for a subwoofer, but it is no good for a full range speaker. The fix is to stuff the pipe. In all of the other geometries, a light stuffing of 0.25 lb/ft^3 is sufficient to produce usable results. Also, in all of the other geometries, only the top half of the pipe is stuffed. This approach is not sufficient for an untapered TL. This is a lightly stuffed TL:



Driver(red) and Port(blue) Output

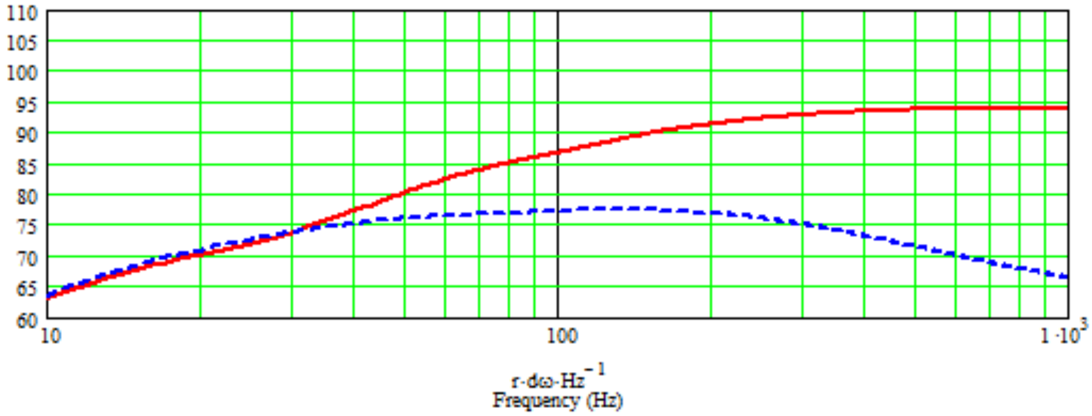


Combined Driver and Port Output(red) and Infinite Baffle Output(blue)

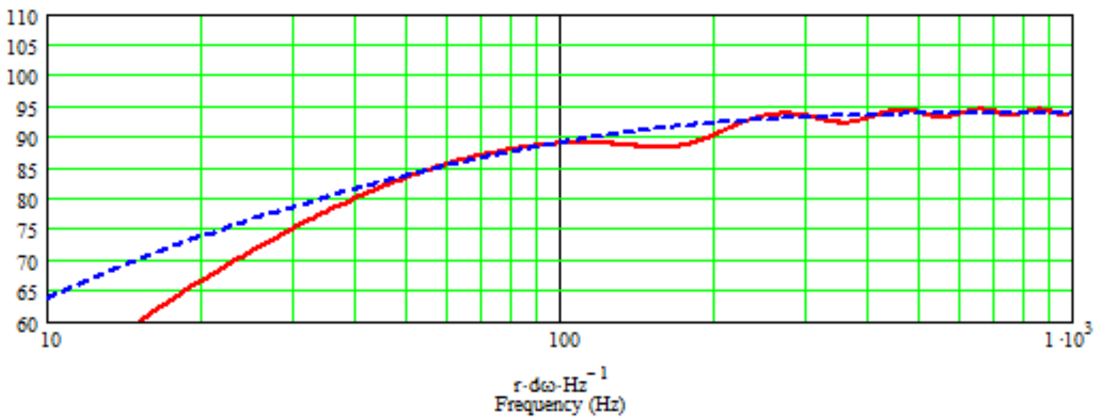


TL Impedance(red) and Infinite Baffle Impedance.(blue)

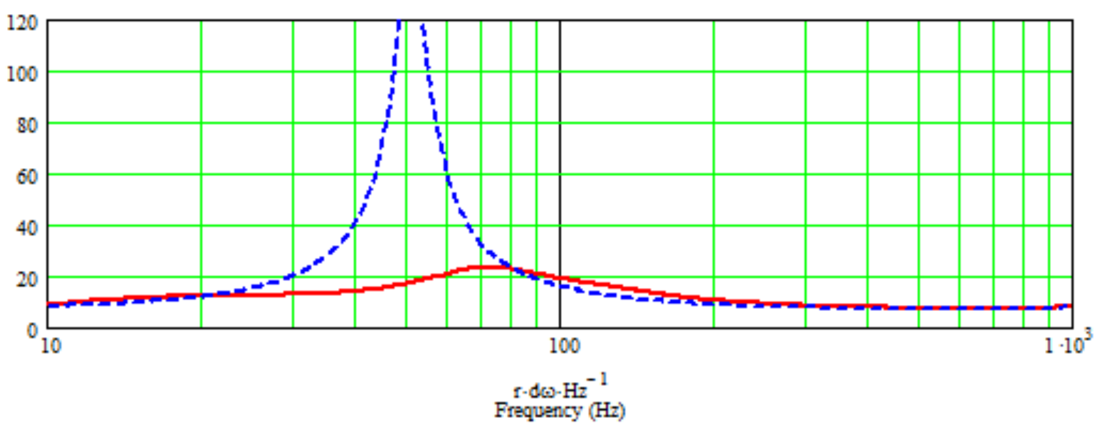
The SPL swings are still as much as 15dB. A straight TL requires a lot more stuffing, and through the entire pipe. In fact, conventional wisdom is that a TL must be stuffed until only one peak appears in the impedance plot. Applying 1 lb/ft^3 stuffing produces this result:



Driver(red) and Port(blue) Output



Combined Driver and Port Output(red) and Infinite Baffle Output(blue)



TL Impedance(red) and Infinite Baffle Impedance.(blue)

What the stuffing has done is strongly suppress the pipe response and cause the combined response to the approximate the infinite baffle curve. Since the port output is so low, this arrangement approaches the theoretical transmission line. There still is some port output, and the driver and port phases still alternate, so there are some wiggles in the combined output. The impedance curve is reduced to a single hump. What has happened is we now have near infinite baffle response in a very finite cabinet. The FE167E is not the ideal driver for this application, but if a high Q_{ts} low f_s driver is used, the results can be very good indeed.

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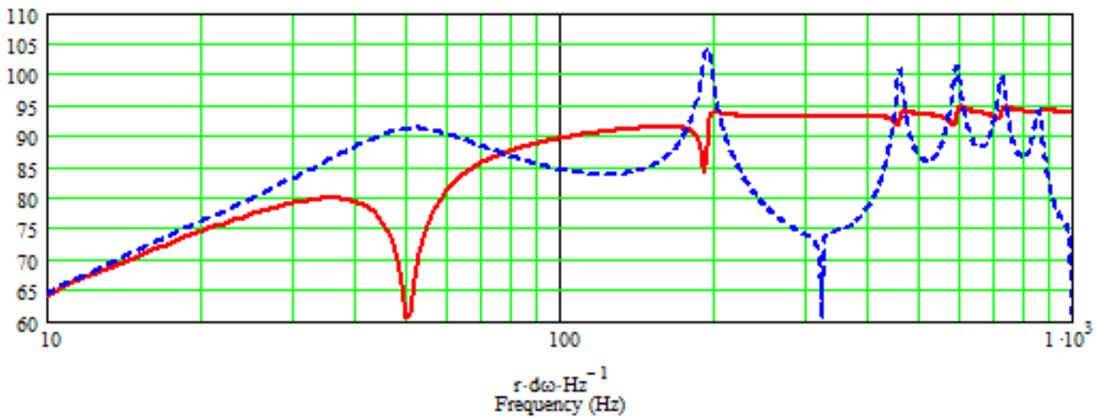
Negative Tapered Transmission Line

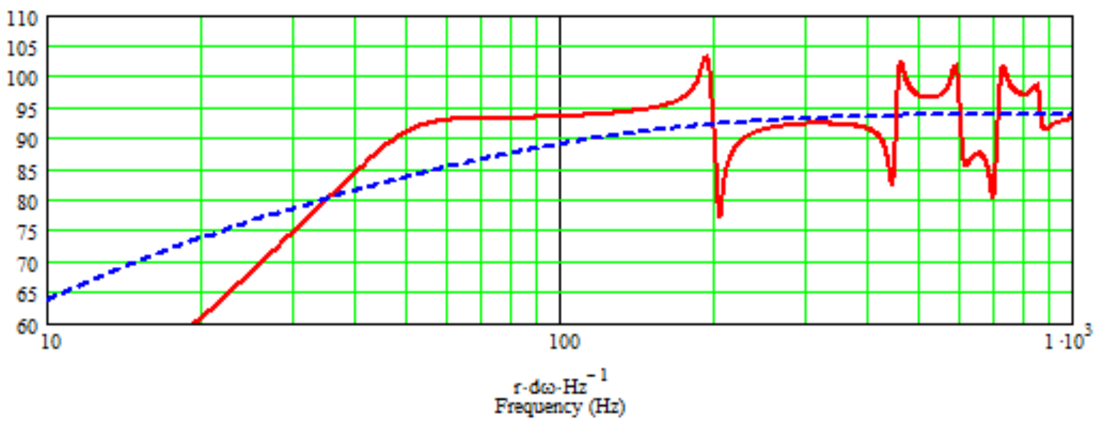
The pipe is 48" long, $S_0$is 3*S_d and S_L is 1*S_d. The driver is at .21*L

Tapered TL's with a large cross section at the top and a much smaller port have been around for a long time. I have added a twist: It has been known for a long time that if the driver is moved to the node of one of the first two overtones, a much smoother output is obtained. The choice of which overtone is suppressed depends on how the pipe will be folded so that the driver may be placed at ear level in the finished speaker. In this example. I have chosen to suppress the second overtone.

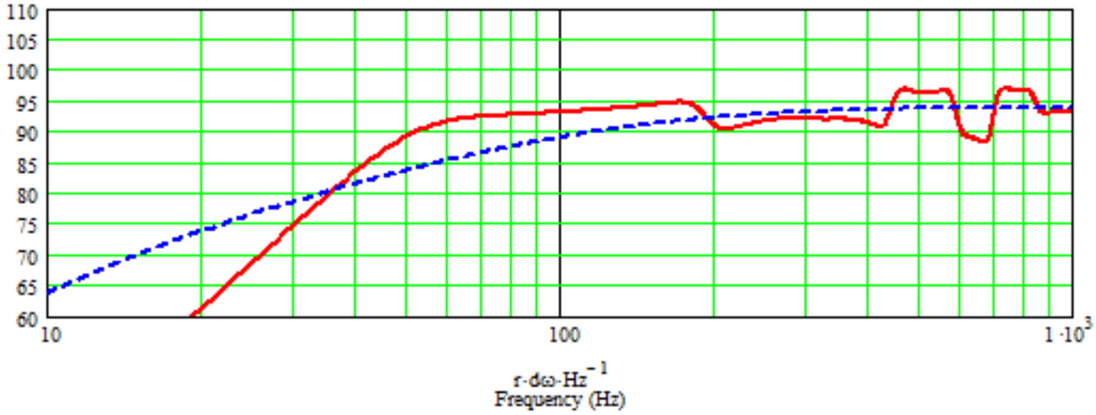
(The impedance graphs and the driver/pipe graphs for the stuffed examples will not be shown from here on as no new information is available. The impedance graphs will show the double peaks and are not suppressed significantly with light stuffing.)



Driver(red) and Port(blue) Output



Combined Driver and Port Output(red) and Infinite Baffle Output(blue)



Stuffed Output(red) and Infinite Baffle Output(blue)

The first thing to note is that the pipe is now substantially shorter for the same tuning. Secondly, but judicious placement of the driver, half of the overtones are now missing. Even with light stuffing, the resulting response is usable. In practice, when the port is placed on the back of the cabinet, the wiggles above 400Hz are less pronounced than predicted.

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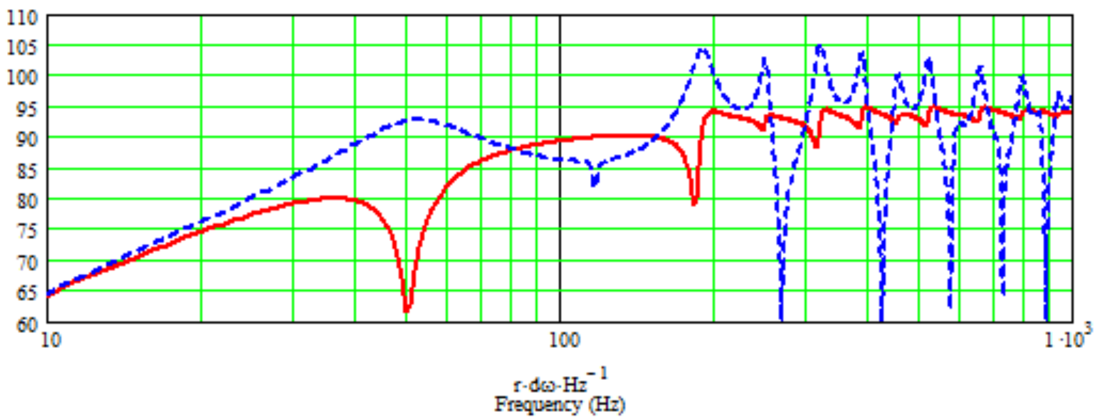
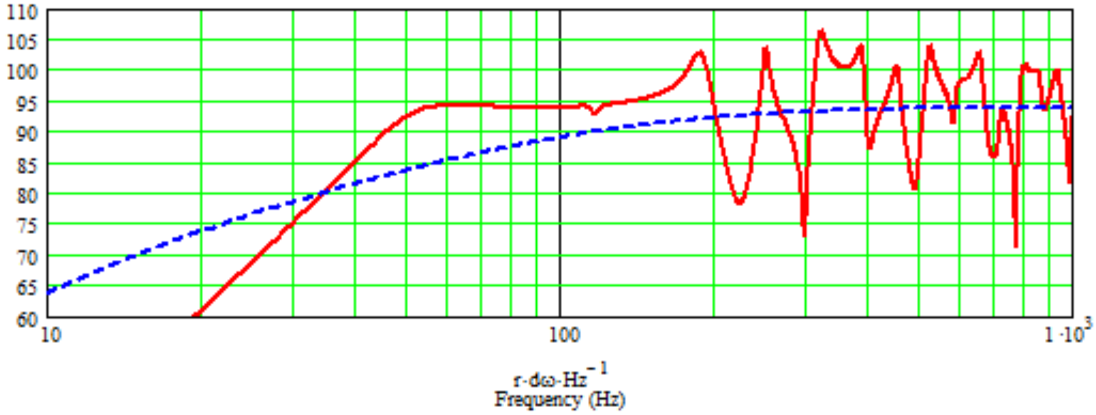
Tapered Quarter Wave Tube (TQWT)

The pipe is 93" long, $S_0$is 0.1*S_d and S_L is 3*S_d. The driver is at 0.45*L.

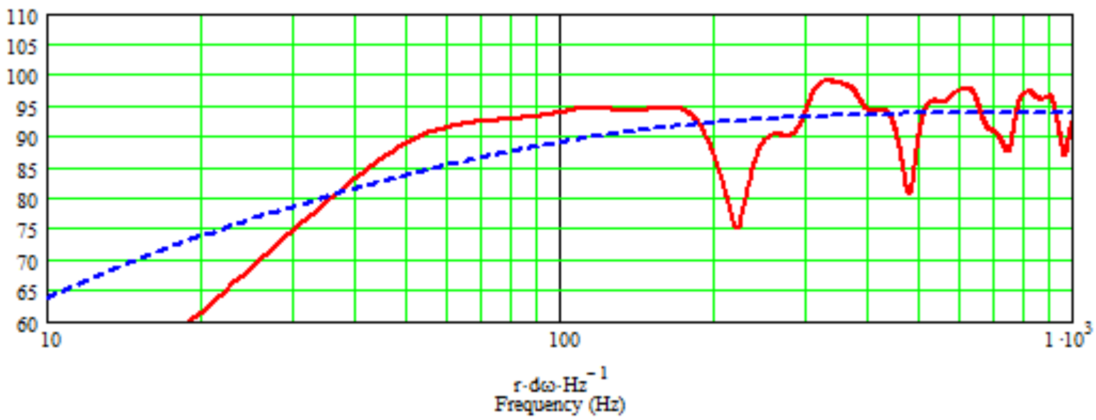
The TQWT, also known as the "Voigt Pipe", has a long and unfortunate history. While the example here is suboptimal, it demonstrates the shortcomings of the vast majority of TQWT designs. TQWT's are long. This example is longer than most because most practical TQWT's have a port area less than the area at the bottom of pipe. Having a reduces port area is a technique called "mass loading". I will discuss this in later examples. The classic "Voigt Pipe" is 72" long. The classic driver position for a TQWT is roughly half way down the pipe. This suppresses the first overtone. (The impedance graphs and the driver/pipe graphs for the stuffed examples will not be shown from here on as no new information is available. The impedance graphs will show the double peaks and are not suppressed significantly with light stuffing.)



Driver(red) and Port(blue) Output



Combined Driver and Port Output(red) and Infinite Baffle Output(blue)



Stuffed Output(red) and Infinite Baffle Output(blue)

It can be seen from the graphs that half of the overtones have been suppressed. However, note that the suppression here is not nearly as effect as in the last example. Even when stuffed, the output remains ragged. While most published designs have a somewhat reduced port area, this is quite typical of their output. The response can be further smoothed by increasing the damping, but what happens is, as in the case of the straight TL above, the output becomes an approximation of the infinite baffle response.

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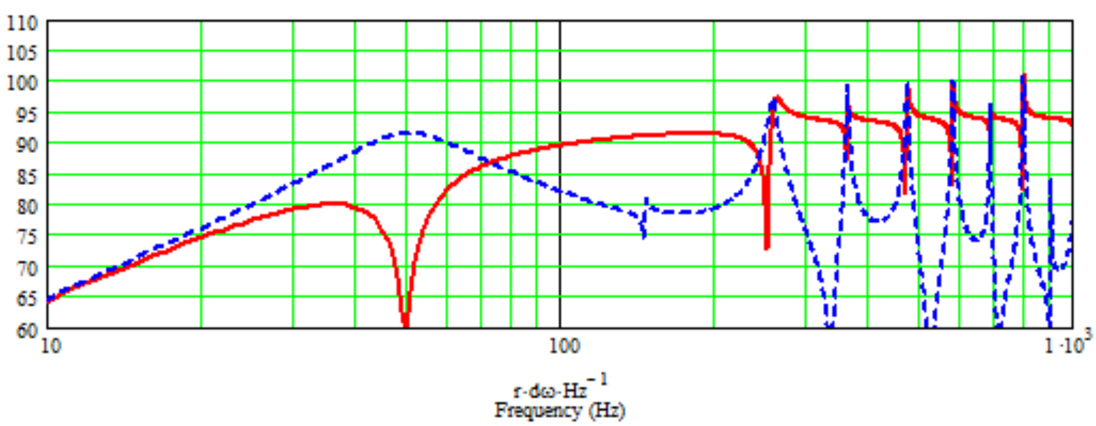
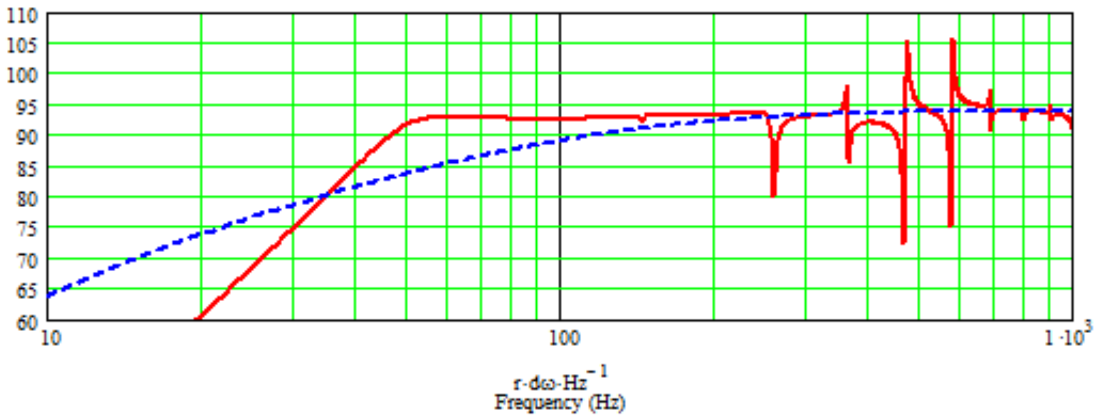
Mass Loaded Tapered Quarter Wave Tube (ML-TQWT)

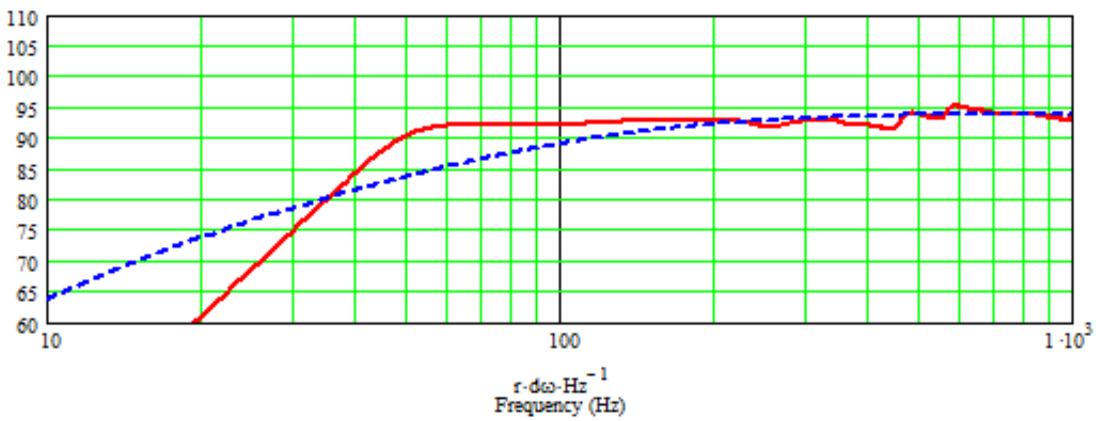
The pipe is 60" long, $S_0$is 0.1*S_d and S_L is 3*S_d. The driver is at 0.57*L. The port is 2" dia x ¾".



Driver(red) and Port(blue) Output



Combined Driver and Port Output(red) and Infinite Baffle Output(blue)



Stuffed Output(red) and Infinite Baffle Output(blue)

The ML-TQWT is only 2/3 as long as the TQWT. Alternating overtones as suppressed much more completely that in the TQWT. All that remains are very sharp spikes that are completely eliminated with light stuffing.

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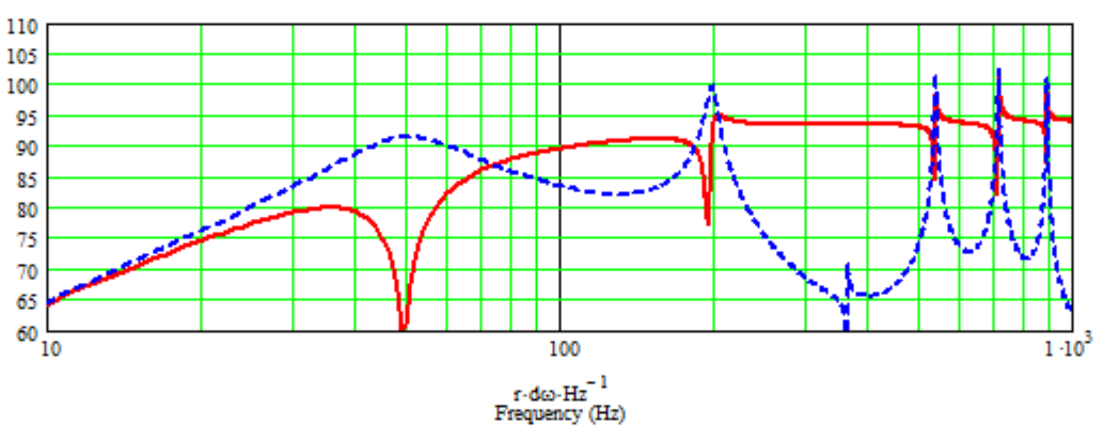
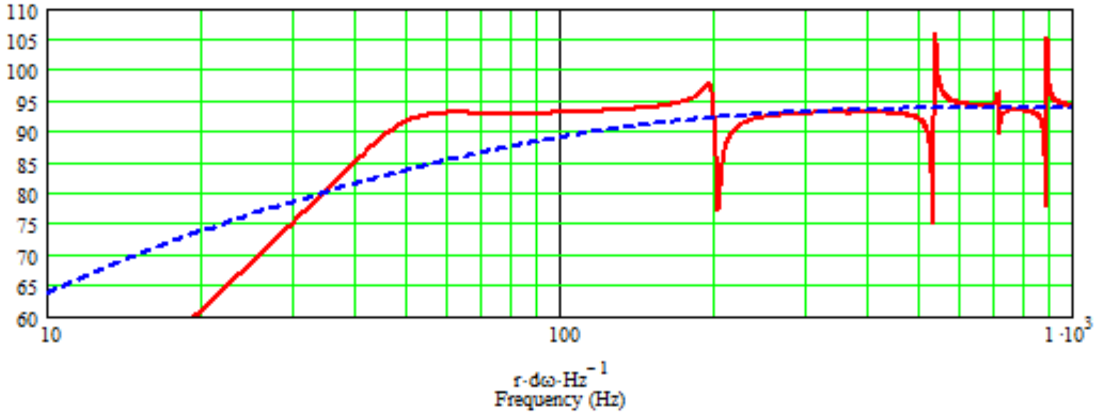
Mass Loaded Transmission Line (MLTL)

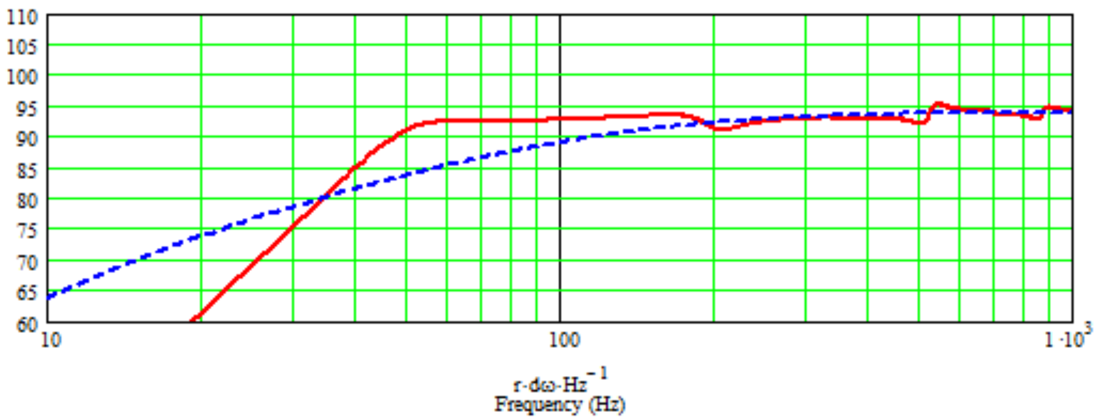
The pipe is 38" long, S_0 and S_L are2.5*S_d. The driver is at 0.25*L. The port is 2" dia x ¾".



Driver(red) and Port(blue) Output



Combined Driver and Port Output(red) and Infinite Baffle Output(blue)



Stuffed Output(red) and Infinite Baffle Output(blue)

The pipe is shorter yet. The driver is positions to suppress the second overtone. The overtones above the first are reduced to spikes, but are basically 20dB below the driver output. Light stuffing produces a very smooth output. This is a very practical configuration. It is not, however the configuration that used for the FT-1600 cabinet. There are other considerations involved in speaker design!

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Observations

This is a summary of the various pipe dimensions. It is ordered by longest pipe to shortest pipe.As a point of reference, the theoretical quarter wave length of 50Hz in free air at 72°F is 67". The modeled TL has an end correction of 2¼", for a modeled lenght of 64¾"

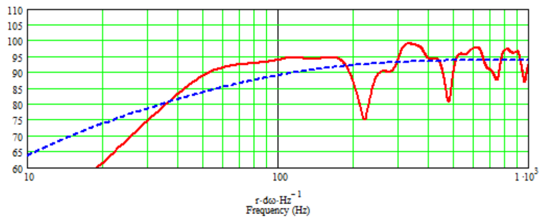
Pipe Geometry Summary

Type	L	S_0	S_L	D	Port
TQWT	93"	$0.1*S_d$	$3*S_d$	.45*L	S_L
TL	62½"	$3*S_d$	$3*S_d$	0.0*L	S_L
ML-TQWT	60"	$3*S_d$	$0.1*S_d$	.57*L	2"x¾"
-ve TL	48"	$3*S_d$	$1*S_d$	.21*L	S_L
MLTL	38"	$2.5*S_d$	$2.5*S_d$	.25*L	2"x¾"

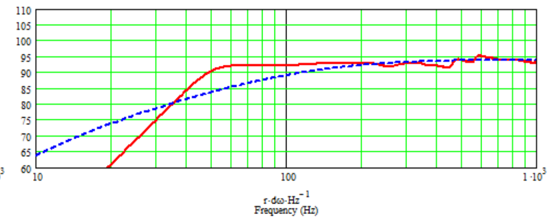
Here are the combined output (stuffed) graphs of the five pipes. On the left are the open

ended pipes, longest to shortest, and on the right, the mass loaded equivalent. There is no mass loaded equivalent to the negative tapered TL. This configuration is already significantly mass loaded and further constricting the port does not normally make sense.

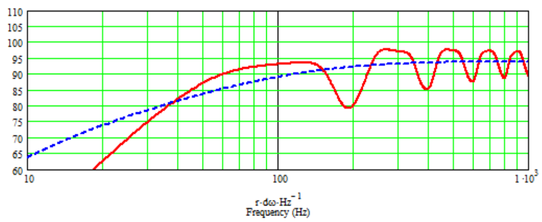
Graphic Comparison



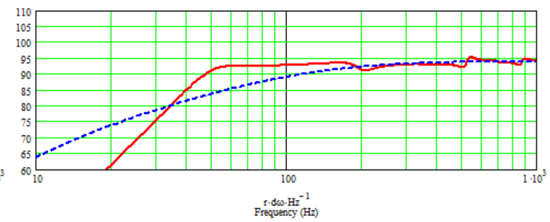
TQWT



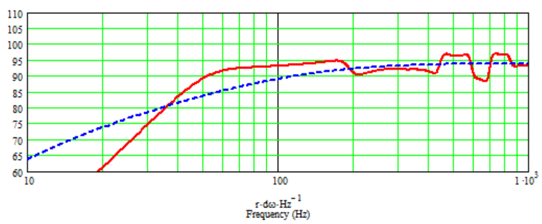
ML-TQWT



TL



MLTL



-ve Tapered TL

- Changing the taper of the pipe changes the length of the pipe to obtain the same resonant frequency. A positive tape, i.e. the pipe is larger at the open end than at the driver end, produces a pipe longer than the free air quarter wave length. A negative taper pipe will be shorter than the free air quarter wave length.
- Mass loading the pipe, that is making the port smaller than the area of the pipe at the open end, shortens the length of the pipe.
- Stuffing the pipe smoothes the output. Light stuffing removes the spikes without affecting the underlying trend. Heavy stuffing will reduce the output more heavily at the bass end than the treble end.

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Driver Suitability

This is not a design tutorial. I present these data to demonstrate what happens as the geometry of the pipe changes. The driver I chose for this demonstration is the Fostex FE167E. It has moderate Q_{ts} and V_{as} which allows it to work in all of the geometries presented. A final, practical configuration will depend entirely upon the parameters of the driver chosen, and that configuration must be optimized for the driver. In generalities, low Q_{ts} drivers ($Q_{ts} < 0.3$) will tend toward TQWT's, while high Q_{ts} drivers ($Q_{ts} > 0.4$) will tend toward negative taper TL's.

The V_{as} of the chosen driver will determine the volume of the pipe. In the examples here, the pipe volume is set to V_{as}, more or less. This is usually a good starting point. However, drivers with extreme V_{as} values can cause problems. Many of the the old legacy drivers have very high V_{as} values and will require huge cabinets. On the other hand, some modern drivers have such low V_{as} values that pipes are not practical.

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Glossary

The nomenclature and conventions surrounding quarter-wave resonators is not well establish, so for the purposes of this web site, I will try to adhere to the following: (To avoid the cumbersome term "quarter-wave resonator", I will call this class of cabinet a "pipe".)

These terms are in common usage and do not necessary reflect technical characteristics:

- ## Transmission Line (TL)

  has either no taper, or is largest at the closed end and tapers down to the vent. The TL normally has the driver very close to the closed end. There may be an expansion chamber immediately behind the driver. The term transmission line comes from electronics where the ideal transmission line does not reflect power back into the source. To make a TL a true transmission line, it must be stuffed sufficiently to completely damp any output from the vent.

- ## Mass-Loaded Transmission Line (MLTL)

  is a term coined by Martin King that applies to a TL terminated by a port that is smaller in area than the area of the pipe at the termination.

- ## Tapered Quarter-Wave Tube (TQWT)

  is the reverse of a tapered transmission line, that is, it is smallest at the closed end tapering up to the open end. The driver is normally set about half way down the tube. The TQWT may also comes in a mass-loaded variety, the ML-TQWT. As with the MLTL, the port area is smaller than the pipe area at the termination.

- ## Voigt Pipe

  is a TQWT that comes to a point at the closed end. It is interesting that this configuration should be called a Voigt pipe, since it appears that Voigt himself did not advocate it. This configuration can have some very bad characteristics that require careful consideration. The most common implementation is the Lowther Club of Norway TQWT, which, unfortunately, is not the best of designs.

Let me establish a convention. A pipe produces resonance at multiples of the fundamental frequency. I will call the fundamental frequency F_1, the first harmonic. The other way way of doing it is to call the fundamental frequency F_0, which leads to confusion when talking about odd and even harmonics. (Harmonic numbers will use a capital "F". Cut-off frequencies, i.e., f_3, f_{10} will use lower case "f".)

Pipe parameters are"

- S_d = Area of the Driver

- S_0 = The area of the closed end of the pipe
- S_L = The area of the open end of the pipe
- A_p = The area of the port
- L = The length of the pipe
- D = The position of the driver (from the closed end as a ratio of length)