

MODELING AND COMPENSATION OF NONLINEAR DISTORTION IN HORN LOUDSPEAKERS

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Abstract

The horn loudspeaker suffers from producing high distortions. Based on an physical model an equivalent lumped parameter circuit is determined for a horn loaded compression driver. From the nonlinear differential equation of this circuit, a compensation algorithm is implemented on a Digital Signal Processor (DSP) for reducing nonlinear distortion caused by the behaviour of air. Test results are given.

1 INTRODUCTION

Horn loaded compression drivers are widely used in the area where high sound level pressures together with good directivity characteristics are needed. Additional advantage is the high efficiency compared to direct radiator loudspeakers. Disadvantages are the higher costs and greater size. Major disadvantage, however, is the production of high acoustical distortions, often worse than 30 percent harmonic distortion. Instead of improving electro–acoustical transduction by mechanical construction means, we connect a nonlinear circuit in series with the loudspeaker to reduce nonlinear behaviour.

In this paper such a compensation circuit will be described which is based on an equivalent lumped parameter model, using the electro–mechano–acoustical circuit based model of [1]–[3]. Our horn loudspeaker is designed for the mid–frequency range i.e. 500–5000 Hz. In this frequency span the wavelength is still large compared to the dimensions of the horn driver and therefore it is allowed to model it using a lumped parameter circuit.

The compensation circuit is implemented on a DSP and is tested in series with the real horn loudspeaker which is schematically depicted in Fig. 1. When we are capable of reducing nonlinear distortion the driver may become more simple constructed which will also reduce the prize.

2 MODELING OF THE HORN LOUDSPEAKER

Two major parts of the horn loudspeaker are the compression driver and the horn as depicted in Fig. 1. The compression driver consists of a glass–fiber diaphragm which is excited by the electrodynamical principle by means of a voice coil in a permanent magnetic field. In front of the diaphragm a phase correction plug is placed to prevent dips in the sound pressure response due to interfering sound waves traveling different paths. However, this correction plug introduces a thin air film between the diaphragm and the plug itself. The thickness of this airfilm is a compromise between nonlinear distortion and desired cavity volume for good (flat) frequency response. Because of high pressures which occur in the compression driver this thin air film will introduce a nonlinearity caused by adiabatic behaviour of air.

In order to determine the total equivalent lumped parameter circuit for the horn loudspeaker, we will first examine the different domains in detail.

2.1 MECHANICAL DOMAIN

The first domain which is considered is the mechanical domain. The electromagnetic driving force F on the diaphragm is formed by the current i in the voice coil multiplied with the force factor: Bl . The

force factor is the product of B : the effective flux density in the air gap and l : the effective voice coil length. From electrical impedance measurements it appeared that the diaphragm does not behave as a rigid damped mass–spring system but as a distributed mass–spring system. This was obtained from resonance frequency shifts due to attachment of an extra mass to the diaphragm with and without the horn mounted on the compression driver. Different authors in the past [4] came up with extensive models for this behaviour. For keeping our model not too complicated we start with a simple expansion of the rigid damped mass–spring model by dividing the diaphragm mass into a central mass portion and an outer mass portion. The physical model of the mechanical domain is given in Fig. 2. The central mass M_1 is coupled through the damped spring system C_{m1} – R_{m1} to the outer mass M_2 which is coupled to earth through the damped spring system C_{m2} – R_{m2} . Because of diaphragm break–up the velocity of the central mass (v) differs from the velocity of the outer mass (v_2). The total resulting velocity is transformed into the acoustical domain and causes a volume velocity $q=S_d \cdot v$. The resulting force F is transformed into a pressure: $p=F/S_d$ with S_d the diaphragm area.

2.2 ACOUSTICAL DOMAIN

For determination of a model for the acoustical part of the horn loudspeaker the horn driver was dismantled. From measurement of the different physical dimensions a first initial guess of the different acoustical parameter values was obtained.

In Fig. 3 the total model for the acoustical domain is depicted. Comparison of measurements and simulations of the equivalent network showed that this model gave the best results. Because it is difficult to determine acoustical parameters from impedance measurements at forehand we must calculate them from known acoustical elements and their formulas.

The pressure p which is created from the movement of the diaphragm is applied to the compliance C_b , formed by the chamber at the back of the diaphragm. The pressure p_g in front of the diaphragm is not the same as the pressure at the back because of a leakage channel between the compression driver housing and the magnet: modeled by a resistance R_b and mass M_b . From the compliance C_g , formed by a thin air film between diaphragm and phase correction plug, narrow channels lead to the throat of the horn. These channels are modeled by the mass M_c and resistance R_c . Finally we have the impedance at the throat of the horn: Z_h .

2.2.1 HORN

To obtain a model which is not too complicated we use the assumption of plane waves propagating in the horn to obtain the well known complex acoustical impedance at the throat of the exponential horn [3]:

$$Z_h = \frac{\rho_o \cdot c}{S_t} \left\{ \sqrt{1 - \left(\frac{f_c}{f}\right)^2} + j \frac{f_c}{f} \right\} \quad (1)$$

With ρ_o the density of air, c the speed of sound in air, S_t the cross–sectional area at the throat of the horn and f_c the cutoff frequency of the horn: below this frequency no sound waves propagate through the horn. To model the impedance at the throat of the horn as a lumped element Eq. (1) can be modeled as a series circuit of a resistance and a negative compliance (capacitor). This modeling is valid for frequencies higher than the cutoff frequency, which in our case is between 450 and 500 Hz.

2.2.2 NONLINEAR COMPLIANCE

A nonlinear element in the acoustical domain is the nonlinear compliance in front of the horn. We assume that the small air volumes in the horn driver behave as adiabatic processes. Therefore the adiabatic compression relation between pressure p_g and specific volume of air V_g is the basis for a relation of the nonlinear compliance:

$$[p_o + p_g(t)] \cdot [V_o + V_g(t)]^\gamma = p_o \cdot V_o^\gamma \quad (2)$$

with $\gamma=1.4$ the adiabatic constant of air, p_o the static pressure and V_o the volume at rest. Using the definition of the mechanical compliance we obtain:

$$C_g(p_g) = \frac{V_o}{p_o \gamma S_d^2} \cdot \left[1 + \frac{p_g(t)}{p_o}\right]^{-\frac{\gamma+1}{\gamma}} \quad (3)$$

This pressure dependent compliance can be approximated by the first three terms of the binomial series. A better approximation however is found by a second order polynomial approximation. This approximation makes a greater error for small pressure variations but still the overall fit is better. A nonlinear model based on this approximation will generate second and third order harmonics. A reduction circuit based on this model will reduce these harmonics.

Note that this nonlinearity is fully modeled by parameters which are known or can be measured from the physical dimensions of the horn driver. It is therefore possible, once we have optimized the linear model and thus have the linear value of C_g , to calculate the nonlinear parameters.

2.3 EQUIVALENT LUMPED ELEMENT CIRCUIT

The electrical domain is the final domain in our consideration. It consists of the well known series circuit of the voice coil resistance R_e and inductance L_e connected to a gyrator with gyrator constant Bl which takes care of the coupling of the electrical domain to the mechanical domain.

Combining all three domains we obtain the total electro–mechano–acoustic equivalent circuit for the horn loudspeaker. This circuit is depicted in Fig. (4).

For all different element values a first initial guess was obtained from electrical impedance measurement with a network analyzer or from measuring physical dimensions. From impedance measurement we obtain the impedance as well as the phase in the frequency span from 400 to 5000 Hz. Initial mechanical parameters values were determined from resonance frequency shifts caused by attachment of an additional mass to the diaphragm. After removing this mass mechanical compliances and damping constants had to be tuned because disassembling the driver caused different strains in the diaphragm.

Using straightforward techniques from network theory we can transform the network of Fig. (4) into an network without the transformer and gyrator. Mechanical and acoustical parameters are transformed into their electrical equivalents which changes the network topology as well. The pressure dependent compliance $C_g(p_g)$ is thereby transformed into a current dependent inductor $L_g(i_g)$.

2.4 SIMULATION

The electrical input impedance of the network of Fig. (4) was simulated using the circuit simulation program HSpice¹. With the circuit simulator we have optimized the parameter values starting from the initial parameter values obtained from impedance and physical dimensions measurement. Parameter optimization on electrical impedance and phase continues until the relative error averaged over all measurement points is smaller than 10^{-3} . Final fitting results are depicted in Fig.(5). From this it is seen that the equivalent circuit provides a good model for the horn loudspeaker in the frequency interval between 1 and 5 kHz. The resonance between 400 and 1 kHz is caused by the series circuit M_c , R_c and Z_h . The discrepancy between measurement and simulation in this span is therefore caused by the use of the simplified model of the input impedance of the exponential horn. Also the leakage through the driver/magnet interface turned out to be greater than calculated; R_b has become smaller. This could be caused by other leakage channels or by absorption caused by the

¹ HSpice is a registered trademark of Meta-Software Inc.

air–viscosity effects.

3 COMPENSATION CIRCUIT

In principle it is of course possible to design a compensation circuit based on the lumped element circuit from section 2 using the Volterra series expansion as was done by Kaizer for a direct radiator loudspeaker [5]. Main disadvantage of this method is the amount of work to determine the Volterra kernels because the lumped element model contains many elements. Also, to obtain convergence at a relative restricted number of coefficients, the use of the Volterra series expansion is restricted to weakly nonlinear systems. The first measurements on the horn loudspeaker have revealed a strongly nonlinear behaviour.

Therefore we will use a more ad hoc method to obtain a compensation circuit. This method was used by Klippel for a direct radiator loudspeaker [6]. From the nonlinear differential equation obtained from the equivalent network the linear and nonlinear parts are separated. From the nonlinear part a compensation circuit is derived, which in series with the real horn loudspeaker, will give a linear response. This compensation circuit is a nonlinear system containing linear frequency dependent parts, and nonlinear frequency independent parts.

The nonlinear compensation filter will predistort the input signal of the horn loudspeaker such that the total system: i.e. the cascade of filter and loudspeaker behaves as a linear system.

The transfer function of the filter is determined from the difference between the nonlinear differential equation and the desired linear differential equation from which we obtain:

$$u_{out}(t) = u_{in}(t) + \frac{1}{M} \cdot \mathcal{L}^{-1}\{H_1(s)\} * (L_g(i_g(t)) \cdot i_g(t)) + \frac{1}{M} \cdot \mathcal{L}^{-1}\{H_2(s)\} * L_{mb} \cdot \frac{d(L_g(i_g(t)) \cdot i_g(t))}{dt} \quad (4)$$

With M the gain of the amplifier which is connected between compensation filter and loudspeaker, u_{out} the output voltage and u_{in} the input voltage of the filter. The inverse Laplace operator $\mathcal{L}^{-1}\{\}$ transforms the impedances into the time domain and the symbol $*$ denotes convolution. $H_1(s)$ and $H_2(s)$ are linear transfer functions containing many pertinent parameters and $i_g(t)$ is the current through $L_g(i_g)$ representing the transformed pressure in front of the diaphragm. Eq.(4) represents the compensation algorithm which is implemented on a DSP. Besides the input signal $u_{in}(t)$ we need the additional signal $i_g(t)$. This signal is synthesized from the input signal by means of linear filtering. Addition and multiplication of two time signals is done using adders and multipliers while convolution with a linear response and differentiation are performed using linear filters.

Digital linear filters are obtained using the bilinear transform while the differentiator was implemented as an equiripple FIR filter using the Remez–Parks algorithm [7].

4 RESULTS

Implementation of the algorithm on a digital signal processor was done using the high level design and simulation package Signal Processing WorkSystem (SPW²). Using this package C–code was generated for a TMS320C30 DSP which is located on a PC–board together with A/D and D/A convertors, clock generation etc.

A sample–frequency of 9250 Hz appeared to be the maximum for the implementation of the algorithm because of hardware problems. This means that reduction of second and third order harmonics is possible up to approx. 1500 Hz.

4.1 SIMULATIONS

Before the compensation circuit was tested in series with the loudspeaker we implemented the model of the horn loudspeaker on the DSP. From this we obtain a qualitative impression of the agreement between distortion measured from the real loudspeaker and the model.

² SPW is a registered trademark of Comdisco Systems, Inc.

The results are depicted in Fig.6 where second and third order harmonics together with the fundamental response of the model as well as from the real loudspeaker are given as a function of the fundamental frequency which is varied between 400 and 1500 Hz. As can be seen from this result the agreement is quite good as we take into account that there has been no optimization of the nonlinear parameters on distortion products. Especially in the frequency span from 600 to 1100 Hz the qualitative agreement of second and third order harmonics is quite good.

The most probable major cause for the discrepancy between the model and the real measured distortion products is that we did not model the influence of the horn on the higher harmonics nor on the fundamental. Considering this we have obtained a fairly good fit using a rather simple model for the horn loudspeaker.

4.2 MEASUREMENTS

Measurements on the horn loudspeaker were performed using a microphone at 2 cm in front of the horn mouth. The loudspeaker was driven at a voltage of $2.5 V_{\text{peak}}$ which gives distortion values up to 30 % around 1500 Hz. Above this frequency distortion rapidly decreases to a few percent so distortion reduction is interesting up to a frequency of approx. 2 kHz. This is mainly caused by the major resonance frequencies which all lie in this span including the resonance frequency of the cavity in front of the diaphragm.

In the upper part of Fig.7 the measured second and third order harmonic distortion with and without compensation circuit are depicted. As we expected from simulations it appears that reduction is the best in the frequency span from 600 to approx. 1050 Hz with a maximum reduction of second order harmonic distortion of approx. 15% around 900 Hz. Simulations already have shown that around this frequency the cavity in front of the diaphragm has its resonance frequency and therefore the distortion has a maximum. Other distortion maxima are not reduced. The maximum around 400Hz is caused by the horn and around 1500 Hz by the mechanical resonances. Both were not included in the nonlinear model so this result is not surprising.

In the lower part of Fig.7 the second and third order distortion reduction at different driving levels are depicted at a fixed frequency of 800 Hz. As we expected the performance of the reduction is limited for lower driving levels because of the nonlinear polynomial approximation we used.

Considering the third order reduction obtained it is clear that it is less than the second. This could have been predicted from the simulation. From Fig.6 it is clear that second order harmonics are better predicted by the model than the third order. This is caused by the low sample frequency and the use of the discrete linear filters near the fold over frequency.

Considering both measured second and third order harmonics it is clear that at some frequency spans the third order harmonics are greater than the second order. From these results it can be expected at forehand that with Volterra modeling of this loudspeaker much terms will be needed before the series will converge.

5 CONCLUSIONS

The main conclusion we draw from this research up till now is that with a relatively simple model it is possible to reduce nonlinear distortion of a horn loudspeaker. Although this is an encouraging result we have not reached reduction of the distortions around 400 and 1500 Hz. If we want to compensate at these frequencies also, extension of the nonlinear model and proper modeling of the horn itself are inevitable.

The sensitivity of the determined nonlinear parameters turned out to be great so proper determination of their value is important.

Further expansion of the nonlinear model will be the first research objective where the maximum number of instructions which can be executed in one sample period by the DSP happens to be a practical limitation of the model expansion.

6 REFERENCES

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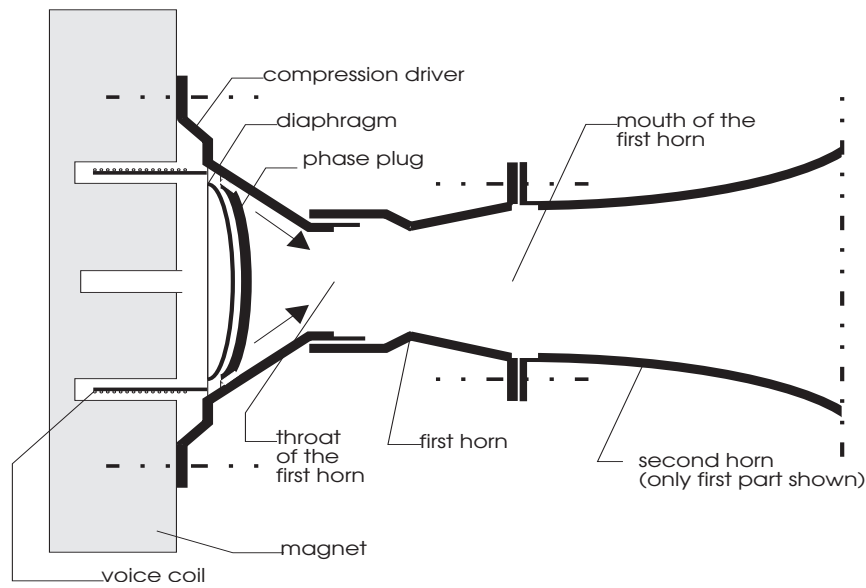


Fig. 1 Cross sectional view of the used compression driver. The diaphragm couples to the throat of the first (small) horn through a small cavity formed by the phase correction plug.

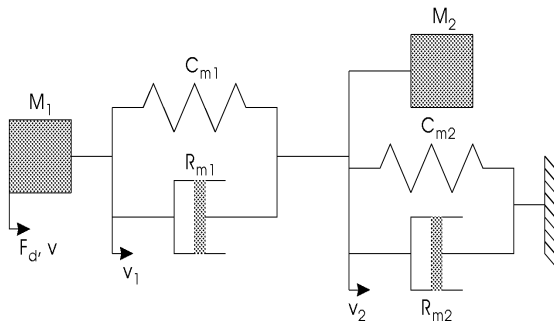


Fig. 2 Physical model of the mechanical part of the horn driver
Driving force F_d represents the electro-magnetic driving force on the diaphragm.

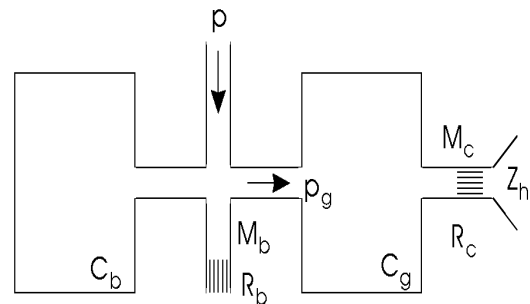


Fig.3 Physical model of acoustical part of the horn loudspeaker.

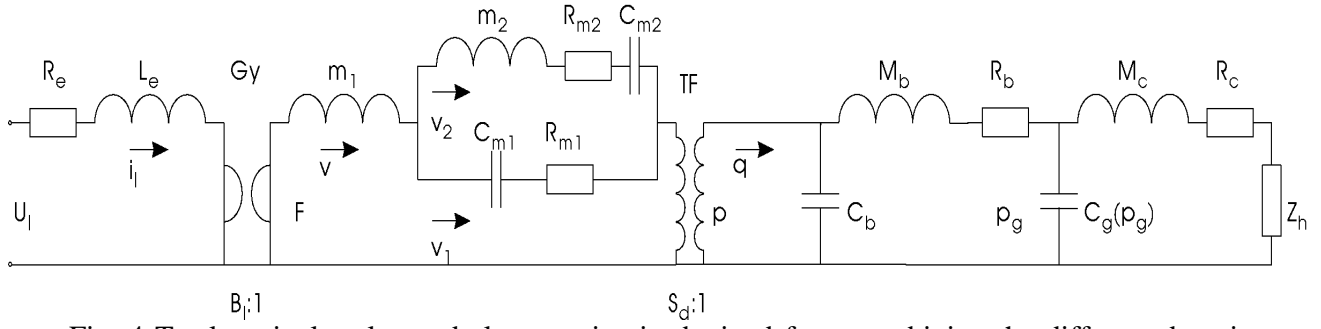


Fig. 4 Total equivalent lumped element circuit obtained from combining the different domains. Nonlinear element is the pressure dependent compliance C_g .

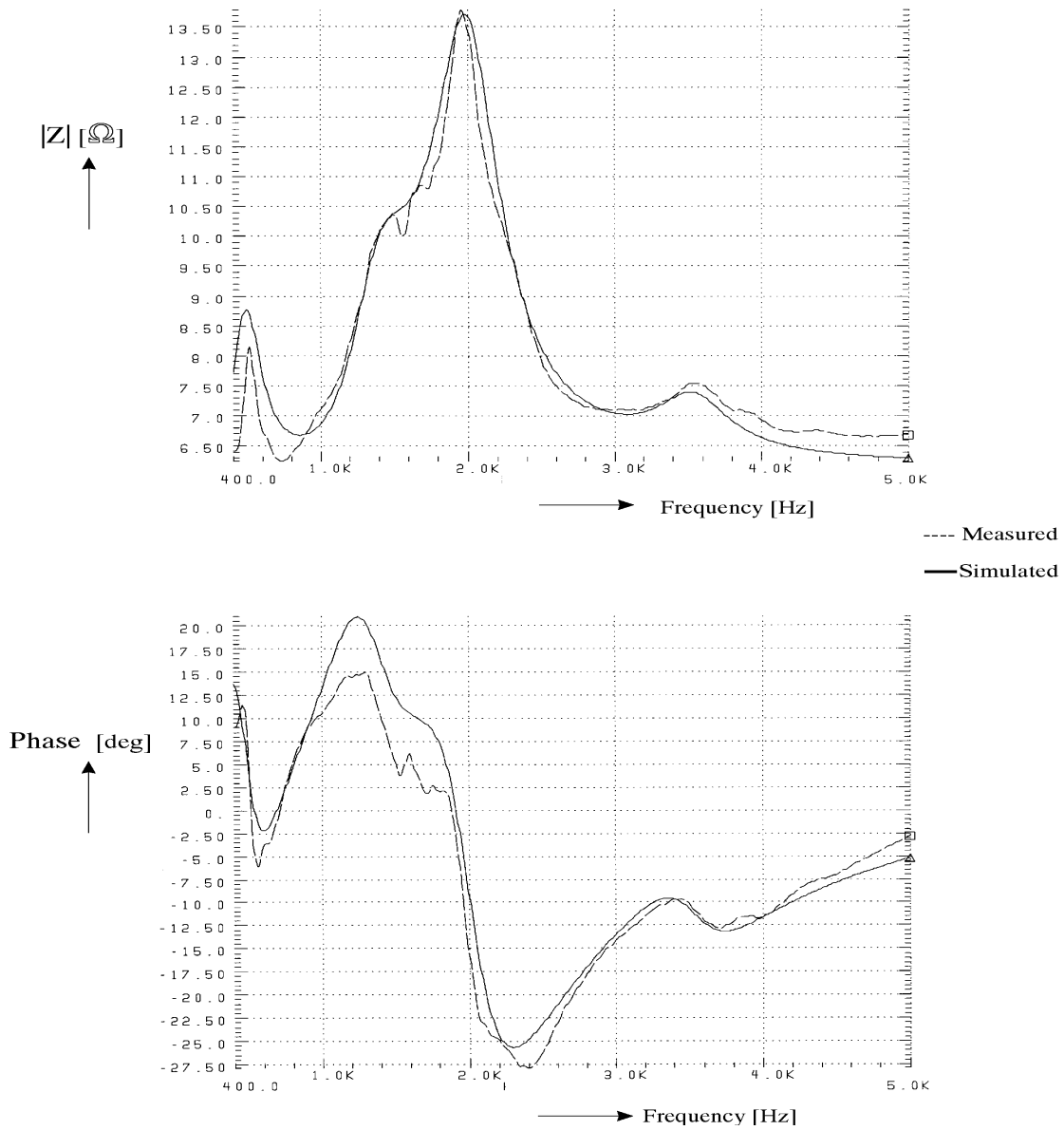


Fig. 5 Simulation results of the input impedance of the equivalent circuit optimized on impedance measurements.

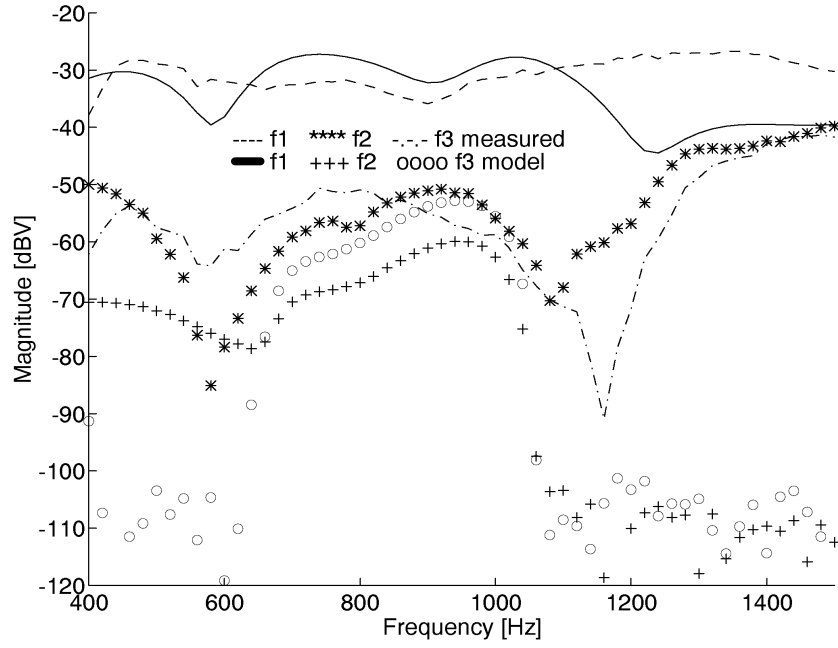


Fig. 6 Measured fundamental (f1), 2nd (f2) and 3rd (f3) harmonics of real horn loudspeaker and from the model on DSP.

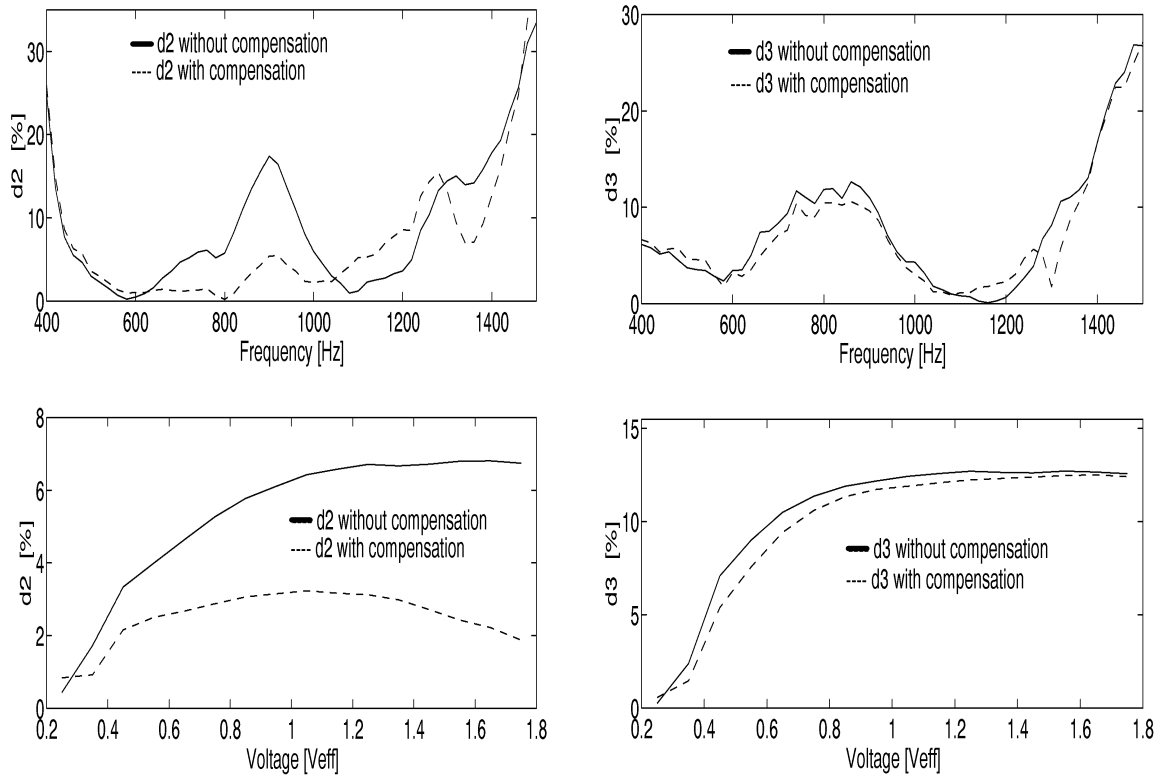


Fig. 7 Measured second and third order harmonic distortion with and without compensation circuit. Upper figures depict measurement with a voltage of $2.5 V_{\text{peak}}$ at the loudspeaker and the lower at different driving levels.