EGO shape optimization of horn loaded loudspeakers

Rick C. Morgans (rick.morgans@adelaide.edu.au) *, Anthony C. Zander (anthony.zander@adelaide.edu.au) and Colin H. Hansen (colin.hansen@adelaide.edu.au)
School of Mechanical Engineering, The University of Adelaide, SA, 5005

David J. Murphy (dmurphy@krix.com.au)
Krix Loudspeakers, Hackham SA, 5163

Abstract. Horn loaded loudspeakers increase the efficiency and control the spatial distribution of sound radiated from the horn mouth. They are often used as components in cinema sound systems where it is desired that the sound be broadcast onto the audience uniformly at all frequencies, improving the listening experience. The sound distribution, or beamwidth, is related to the shape of the horn and can be predicted by numerical methods, such as the boundary element or source superposition method, however the cost of evaluating the objective function is high. To overcome this a surrogate optimization method called Efficient Global Optimization (EGO) was used with a spline based parameterization to find the shape of the horn that gives a frequency independent beamwidth, thus giving a high quality listening experience. *(Submitted for the SMSMEO-06 special issue).*

Keywords: Horn Loaded Loudspeakers, Constant Beamwidth, Surrogate Optimization, Efficient Global Optimization

1. Introduction

The engineering design of consumer and industrial products is moving from simple analytical and heuristic techniques to using very complex or computationally expensive numerical calculations, often embedded in a numerical optimization routine. This paper describes one such application in engineering design, the shape optimization of a horn loaded audio loudspeaker to improve its sound quality. A numerical technique called the source superposition method, similar to the boundary element method, has been shown to produce accurate simulations of horn loaded loudspeakers. With the choice of an appropriate objective function to measure sound quality and an appropriate shape parameterization, a frequency independent or constant beamwidth horn design can be found that provides position independent frequency response for the listener, more accurately reproducing the intended sound.

Although the source superposition method is very computationally efficient compared to other methods, performing calculations across a

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range of frequencies makes it relatively computationally expensive in this application, and derivatives of the objective function with respect to the input variables are difficult to calculate because of numerical noise. Hence a surrogate optimization technique called Efficient Global Optimization (EGO) is used. This method requires no knowledge of the derivative of the objective function with respect to the input variables, and is suitable for the current application.

The structure of this paper is as follows: a brief description of horn loaded loudspeakers and their use in the cinema industry is given as background to the application; the measure of sound quality used in the optimization (the objective function) is discussed; the geometry parameterization is described; an overview of the numerical models used to calculate the objective function is given; a description of the EGO optimization method is provided; the results of the shape optimization are given; and finally the work is summarized and conclusions are drawn.

2. Horn loaded loudspeakers

The aim of sound reproduction systems in cinemas is to provide a high quality listening experience, accurately reproducing the recording for any listener in the audience. The horn loaded loudspeaker is a component often used in cinema sound systems and in related live sound reinforcement systems. This device is used because it is an efficient audio transducer, and provides some control over the spatial distribution of sound away from the horn mouth. The sound distribution, or beamwidth, is related to the shape of the horn and it is critical for listening quality that the sound be distributed evenly onto the audience at all frequencies.

Figure 1 shows a commercially available cinema loudspeaker system. A horn loaded loudspeaker broadcasts the high frequency content (generally above $500Hz$) and is mounted on top of a low frequency direct radiator loudspeaker. The system is located behind the cinema screen.

A schematic of a horn loaded loudspeaker system is shown in Figure 2, which consists of two main components, a compression driver and a horn flare. The source of the sound, the compression driver, consists of a small (usually titanium) diaphragm driven by a conventional electromagnetic drive (voice-coil and magnet) positioned in front of an abrupt change in cross sectional area. The flare changes the cross sectional area gradually from the throat through to the mouth of the horn. The shape of the horn flare controls the spatial distribution of sound, and the horn flare (or just *horn*) is the component considered in this study.

Figure 1. Commercial cinema loudspeaker (Krix Loudspeakers, 2007).

The distribution of the sound field in front of the horn varies with the frequency of excitation, and a measure of this variation in the far field of the horn (over the audience) is needed. The beamwidth, or coverage angle in a plane, defined as the "angle formed by the -6dB points (referred to the on-axis reading) and the source center" is used. Specification of the beamwidth variation with frequency for both the horizontal and vertical planes has long been industry practice (Davis and Davis, 1997) and more recently and importantly has been used by Lucasfilm, an important industry body, in the specification of sound quality requirements for cinema loudspeaker systems (THX, 1996). In the case of cinema audio, it is critical to the listening experience that the sound be distributed evenly onto the audience at all frequencies (frequency independent or constant beamwidth), and hence beamwidth is the principle measure adopted to assess sound quality.

The optimization of horn loaded loudspeakers to produce a desired outcome has been attempted previously. Examples include an optimization of the beamwidth of a horn loaded woofer using a 3D BEM (Miccoli, 1999), the optimization of the frequency response of a horn loaded tweeter using axisymmetric BEM (Henwood, 1993; Geaves and Henwood, 1996), and most recently FEA to optimize the frequency

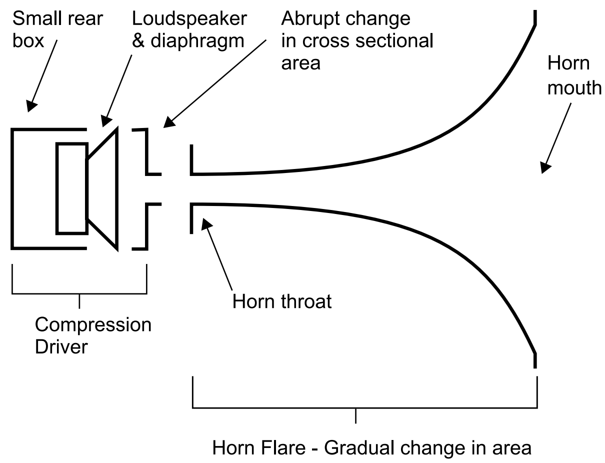

Figure 2. Schematic of horn loaded loudspeaker.

response of a planar horn (Bängtsson et al., 2003). None of these methods have been applied to optimizing beamwidth for the type of horns used in cinema loudspeaker systems.

3. Measures of audience sound quality

For any optimization, an objective function describing the relative merit of the current solution must be calculated. The stated objective for designing horn loaded loudspeakers to improve audience sound quality is a smooth, frequency independent beamwidth. Defining,

$$\Phi_1 = \texttt{mean}\left(\mathcal{B}\left(\mathbf{f} \geq f_{min}\right)\right) \quad (1)$$

$$\Phi_2 = \texttt{std}\left(\mathcal{B}\left(\mathbf{f} \geq f_{min}\right)\right) \quad (2)$$

$$S = \frac{\Phi_2}{\Phi_1} \quad (3)$$

where $\texttt{mean}(\mathbf{x})$ and $\texttt{std}(\mathbf{x})$ are the mean and standard deviation of vector $\mathbf{x}$, respectively, S is an objective measure of the beamwidth smoothness, and $\mathcal{B}(\mathbf{f})$ is the vector of beamwidths calculated over a range of frequencies described by the vector $\mathbf{f}$. The operator $\mathbf{f} > f_{min}$ selects only those frequencies above f_{min}. This operator is required because when the wavelength of sound is large in comparison to the size of the horn (i.e. low frequencies) then the horn has no influence on the sound and the radiation pattern is omnidirectional. The smaller the value of S, the smoother the beamwidth over the range of frequencies considered, so the objective of the optimization can be written as,

$$\min S \quad (4)$$

with an optional equality constraint

$$\Phi_1 = \mathcal{B}_{nom} \quad (5)$$

where $\mathcal{B}_{nom}$ is the nominal (or desired) beamwidth.

Figure 3 shows a typical calculation of beamwidth for a small horn. The solid black line is the beamwidth evaluated at a $50Hz$ frequency spacing, ranging from 300 to $12000Hz$. This fine frequency spacing is inappropriate for the numerical calculations used in the optimization due to computational cost, and the beamwidth was calculated at a coarse $400Hz$ frequency spacing (the vector $\mathbf{f}$). The black circular markers show the values of the beamwidth $\mathcal{B}(\mathbf{f} \geq f_{min})$ at these frequencies. The cutoff frequency $f_{min} = 3000$ avoids the omnidirectional nature of the sound field at low frequencies. The dashed line is the value of Φ_1, the mean value of the beamwidth and a value of S, the objective function is reported.

4. Numerical model

The source superposition technique (Koopmann and Fahnline, 1997) is a numerical method that can be used to solve the acoustic pressure field generated by, and acoustic power radiated from, arbitrarily shaped surfaces. It is similar to a Boundary Element Method, but cannot be

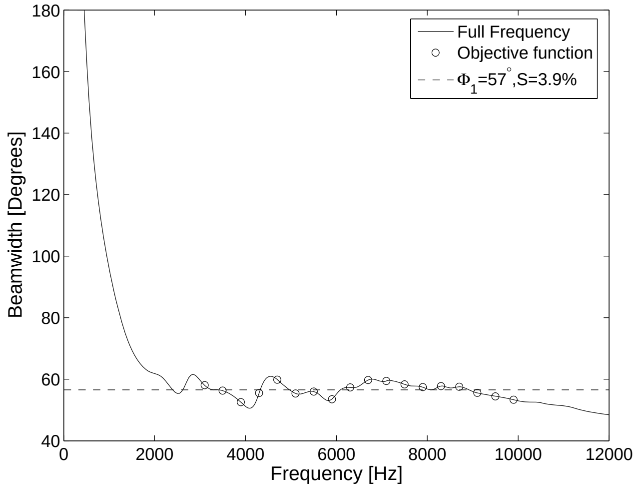

Figure 3. Typical beamwidth calculation for a small horn.

classified as such because it does not directly discretize and solve the Kirchoff-Helmholtz equation (Morse and Ingard, 1986). The technique is fully three dimensional, and not limited to a one dimensional approximation such as those traditionally used to model horns (Holland et al., 1991; McLean et al., 1992; Mapes-Riordan, 1993), which have been shown to be invalid above a certain limiting frequency (Morgans et al., 2005). An extensive review of the literature has been unable to find any previous application of the source superposition technique to horn modeling, although it is an ideal candidate for this because it is able to accurately model the far-field pressure with a limited number of elements per wavelength (Morgans et al., 2004). The technique can also efficiently model "thin" surfaces such as the surface of horn loudspeakers. For further details of the method see Koopmann and Fahnline (1997) and Morgans et al. (2005).

A simple axisymmetric horn has been manufactured to allow experimental validation. This horn, shown in Figure 4, has a 2 inch (50 mm) diameter throat, an 11 inch (280 mm) diameter mouth with a 1 inch (25 mm) flange. It is 9.25 inches (235 mm) in length, and consists of two conical horns joined together, and hence is known as a two step conical horn. Figure 5 shows a quarter symmetric mesh used to calculate the sound field using the source superposition technique.

Excellent agreement between the numerical results and experimental measurements was obtained as shown in Figure 6. In the current work, only axisymmetric simulations are performed, however the methods used can be extended to 3 dimensional calculations as required by industry.

Figure 4. Experimental two step conical horn.

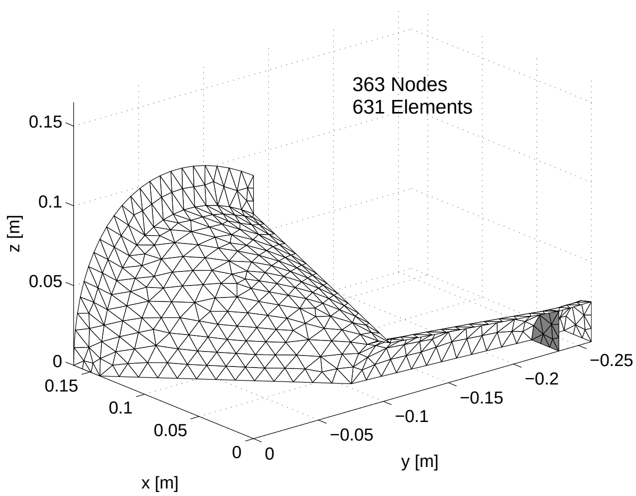

Figure 5. Quarter symmetric source superposition mesh.

5. Geometry parameterization

A spline based parameterization is used to define the horn geometry. The spline starts at the horn throat, at diameter $D_t = 50$ mm (2 inches), and finishes at the mouth, at diameter $D_m = 165$ mm and length $L = 235$ mm. A parametric cubic spline is fit between the start and end points, with two intermediate points to control the shape of the curve. Figure 7 shows the range of geometries possible with this parameterization, with the control points shown as white circles. The small black dots show the lines that fix the position of the control points, with the first control point located a fraction $x1$ along the length

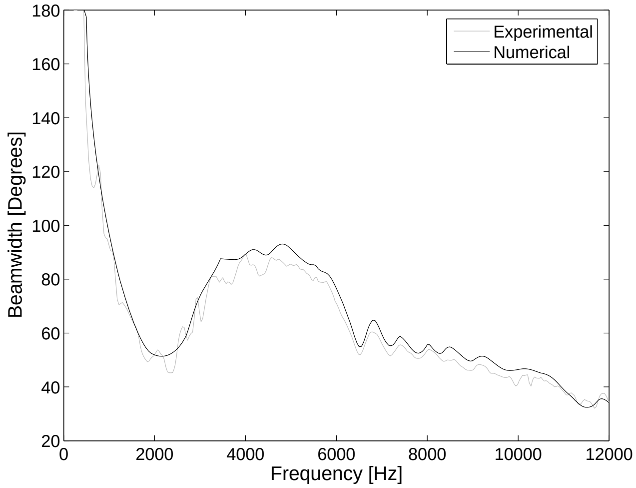

Figure 6. Comparison of measured and calculated beamwidth for the source superposition technique.

of the first line, and similarly for $x2$ the position of the second control point. For further details refer to Morgans et al. (2006).

6. Optimization method

Once the objective function and geometry parameterization are specified, an optimization routine can be called to systematically vary the input parameters until the desired characteristics of the sound field are achieved by minimizing S. Standard gradient based optimization methods such as Sequential Quadratic Programming (SQP) (Schittkowski, 1985) are local optimization methods and as such often have to be run many times from different starting positions to avoid local minima, and even then a globally optimum solution is not guaranteed. In addition, gradient information in the form of the derivative of the objective function with respect to the input parameters is required. In the horn loaded loudspeaker application, gradient calculation is difficult for a number of reasons: no simple analytical gradient calculation is possible; and a finite difference approximation to this gradient is problematic because of the discrete nature of the meshing used in the source superposition method where a small change in horn profile could lead to a jump in the objective function. A finite difference gradient can also require many calls to the function making it relatively computationally expensive. Accordingly, a gradient free surrogate based global optimization technique is preferred, such as the Efficient Global Optimization (EGO) technique (Schonlau, 1997; Jones et al., 1998).

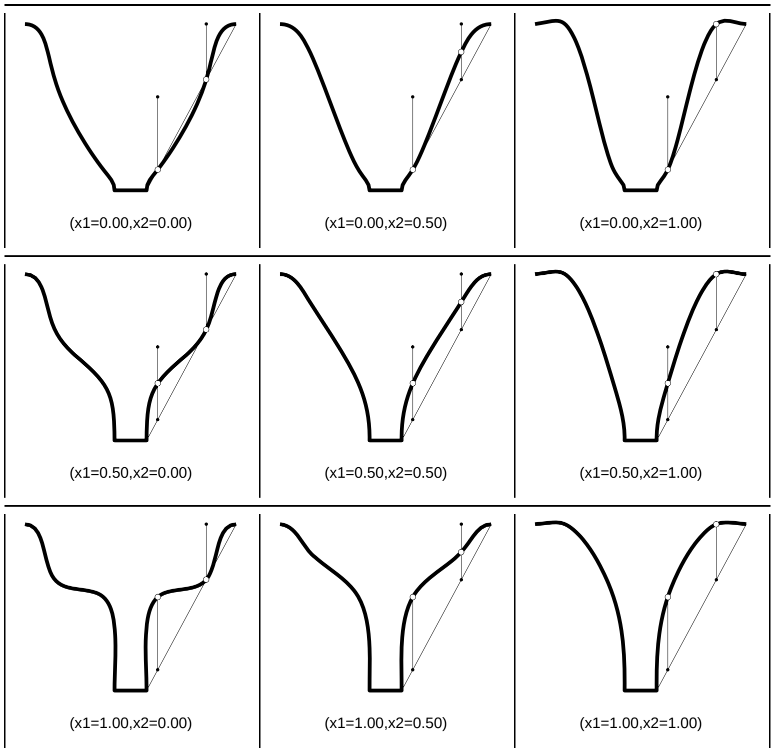

Figure 7. Variation possible in spline horn geometry. Parameters vary between upper and lower bounds, $0 \leq x1 \leq 1$ and $0 \leq x2 \leq 1$.

The EGO technique proceeds as follows. A number of different sets of input parameters are randomly generated to give a representative sample over the range of potential solutions. Here the random samples are generated by Improved Hypercube Sampling (IHS) (Beachkofski and Grandhi, 2002), which attempts to generate a space filling design, but any suitable design of experiments method could be used.

The objective function is then evaluated for each set of input parameters and a surrogate model is fitted to the objective function. This surrogate model describes both the variation of the mean value of the objective function between the sample points and the uncertainty between them, and is much less computationally expensive to evaluate that the original objective function. In this application, a Kriging technique is used (Lophaven et al., 2002). Kriging techniques, developed in the geostatistics and spatial statistics fields, fit a surface to a set of data points values. It models the variation of the unknown function as a constant value plus the variation of a normally distributed stochastic variable. It is essentially a method of interpolation between known points that gives a mean prediction ($\hat{y}(x)$) in addition to a measure of variability of the prediction ($s(x)$, the estimated standard deviation). Another appropriate optimization technique such as SQP, simulated annealing (Ingber, 1993) or the DIRECT method (Finkel and Kelley, 2004) is then employed to find the next best place to sample for a minimum objective function. The secondary objective function used in this application is the Expected Improvement ($\mathbf{E}[I]$) objective function. The improvement function (I) is defined as the improvement of the current prediction, $\hat{y}(x)$, at point x over the minimum value of the current set of samples, y_{min}, i.e.

$$I = \max\left(y_{min} - \hat{y}(x), 0\right) \tag{6}$$

The expected improvement, defined as the expectation of the improvement, is given by

$$\mathbf{E}[I] = \left(y_{min} - \hat{y}(x)\right) \mathrm{CDF}\left(\frac{y_{min} - \hat{y}(x)}{s(x)}\right) + s(x)\,\mathrm{PDF}\left(\frac{y_{min} - \hat{y}(x)}{s(x)}\right) \tag{7}$$

Where $\mathrm{CDF}(x)$ is the standard normal cumulative density function, and $\mathrm{PDF}(x)$ is the standard normal probability density function. The point at which the value of the expected improvement is maximized gives the best point at which to calculate the true objective function. The Expected Improvement is constructed so as to search for both local and global minima (Schonlau, 1997; Jones et al., 1998). The surrogate model is then updated to include the newest sampled point, and the operation repeated until the sampling point does not change and the global minimum of the objective function has been found.

One advantage of the EGO method is that it requires a minimal number of objective function evaluations, and most of the optimization is done on the computationally cheap surrogate. This makes the method very efficient when the objective function is computationally expensive, as is the case in the current application.

7. Results

The results of the EGO optimization of S for the horn geometry described in Section 5 appear in Figure 8. A contour of the Kriging surrogate mean predictions of S is shown, along with the positions of the EGO sample points. The 25 circular markers show where the initial points are sampled, and the 25 square markers show the sample points chosen by the Expected Improvement function, balancing both local and global optimization. A convergence to the global minimum can be seen with repeated sampling (many square markers) around the global minimum (diamond marker) at $x1 = 0.49$ and $x2 = 0.69$. Figure 8 indicates that the global minimum of an unconstrained 2 parameter optimization of S can be found within 50 function evaluations with reasonable certainty, and other optimizations produce similar results. The value of S does change with the frequency spacing chosen for $\mathbf{f}$, but it was found that the optimization converged to similar horn profiles for frequency spacings between 50 and $400Hz$. The horn profile corresponding to the global minimum is shown in Figure 9, and the beamwidth calculated from the optimial horn profile is shown in Figure 10. A nominal beamwidth of $47.1°$ with parameter $S = 3.1\%$ is achieved.

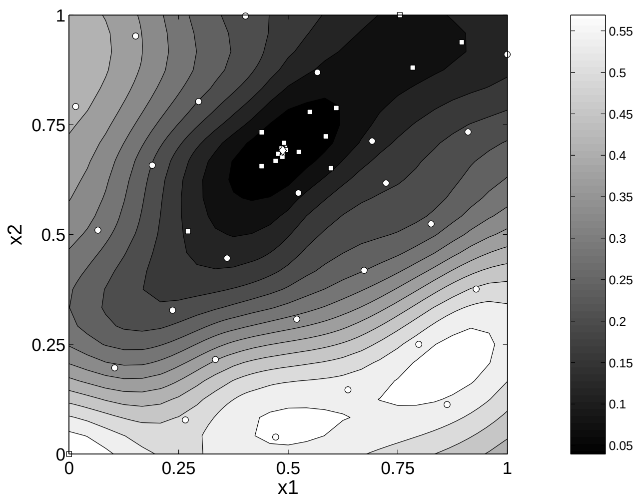

Figure 8. Optimization trajectory for the 2 parameter spline horn geometry.

Comparing Figure 10 with the beamwidth of the experimental horn in Figure 6 it can be seen that a constant beamwidth with frequency has been achieved above a certain limiting frequency. The optimization had no constraint on the value of Φ_1, the mean value of the beamwidth. The inclusion of the desired mean beamwidth as a constraint would be beneficial for the automated design of horns but because the current optimization does not include horn length and mouth as parameters, but as fixed values, achieving an arbitrary mean beamwidth is physically impossible due to geometric constraints. It would be possible to include these extra parameters in a further optimisation, and other methods

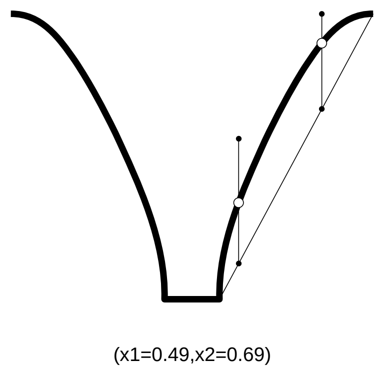

Figure 9. Profile of optimal 2 parameter spline horn geometry.

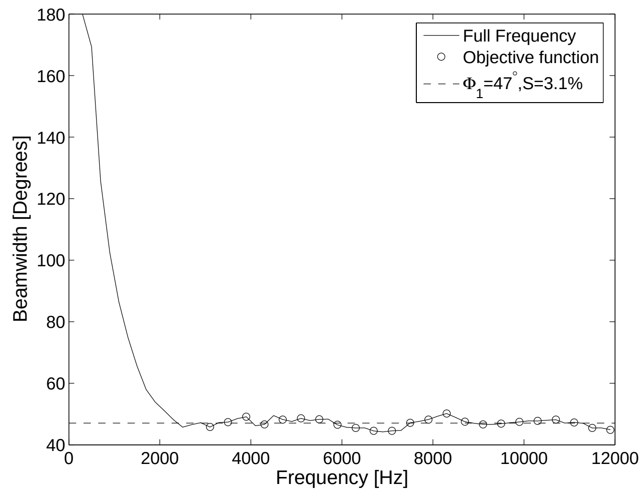

Figure 10. Beamwidth for optimal 2 parameter spline horn geometry.

to overcome this are discussed in Morgans (2005) and Morgans et al. (2006).

8. Summary and Conclusions

This paper has described horn loaded loudspeakers and their application in the cinema industry. It has introduced the concept of beamwidth as a measure of the uniformity of the sound field over the audience, and discussed the need for a frequency independent or constant beamwidth to improve the sound quality for the listener. It has discussed one objective function that succinctly quantifies the aims of a horn designer, and has described a flexible horn geometry based on a parametric cubic spline with two controlling parameters.

Of the many numerical techniques available the source superposition technique has been shown, by comparison with experiments, to be a good choice for modeling the sound field radiated by horns. It is capable of reproducing the sound field generated by a small horn loaded loudspeaker from a specification of the horn geometry, and the accuracy of the reproduction is adequate for design purposes within the required frequency range.

A fast and reliable gradient free optimization technique for expensive objective functions, Efficient Global Optimization, has been introduced, and results of the shape optimization show that it is capable of producing a horn shape that provides a constant beamwidth above a certain limiting frequency. The method can easily be extended to both the specification of a desired mean beamwidth, and to 3 dimensional calculations, as required by industry. Overall, the Enhanced Global Optimization technique is a robust design method for horn loaded loudspeakers.

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