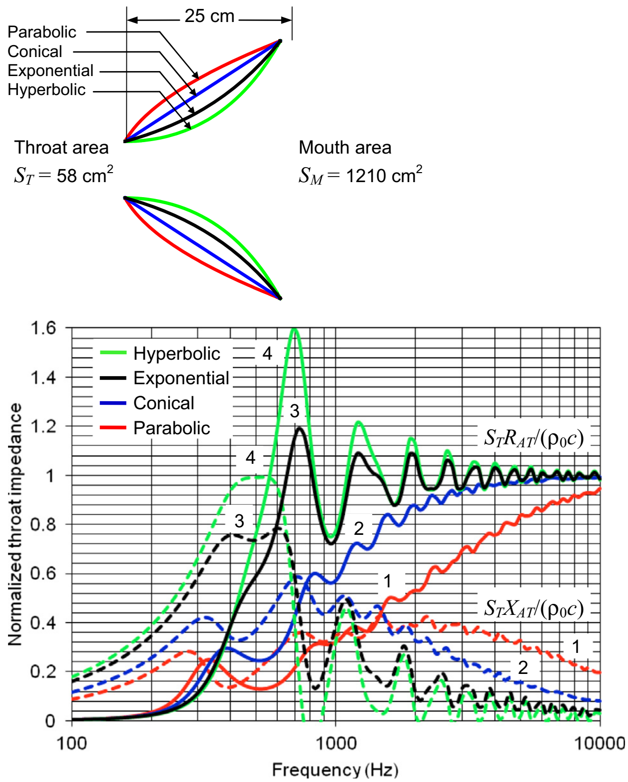

Figure 9.10 Plot of normalized throat impedances for finite parabolic (1), conical (2), exponential (3), and hyperbolic (4) horns using Eq. (9.64) and Eq. (13.116) for $Z_{AM} = Zs/S_M$, assuming termination in an infinite baffle. Real impedances $S_T R_{AT}/(\rho_0 c)$ are represented by solid curves and the imaginary impedances $S_T X_{AT}/(\rho_0 c)$ are represented by dashed curves. The value of α for the hyperbolic horn is ½. The cutoff frequencies of the parabolic, conical, exponential, and hyperbolic horns are 1182 Hz, 792 Hz, 337 Hz, and 399 Hz respectively.

Finite conical horn

The matrix elements are given by

$$a_{11} = \sqrt{\frac{S_M}{S_T}}\left(\cos kl - \frac{1}{kx_M}\sin kl\right) \tag{9.71}$$

$$a_{12} = j\frac{\rho_0 c}{\sqrt{S_T S_M}}\sin kl \tag{9.72}$$

$$a_{21} = j\frac{\sqrt{S_T S_M}}{\rho_0 c}\left\{\left(\frac{1}{kx_M} - \frac{1}{kx_T}\right)\cos kl + \left(1 + \frac{1}{k^2 x_M x_T}\right)\sin kl\right\} \tag{9.73}$$

$$a_{22} = \sqrt{\frac{S_T}{S_M}}\left(\cos kl + \frac{1}{kx_T}\sin kl\right) \tag{9.74}$$

where S_T is the area of the throat, S_M is the area of the mouth, and the length l of the horn from the throat to the mouth is given by $l = x_M - x_T$, so that $x_T = l/(\sqrt{S_M S_T} - 1)$. The throat impedance of a finite conical horn is plotted in Fig. 9.10.

Finite exponential horn [13]

The matrix elements are given by

$$a_{11} = \sqrt{\frac{S_M}{S_T}}(\cos(kl\cos\theta) - \tan\theta\sin(kl\cos\theta)) \tag{9.75}$$

$$a_{12} = j\frac{\rho_0 c}{\sqrt{S_T S_M}}\sec\theta\sin(kl\cos\theta) \tag{9.76}$$

$$a_{21} = j\frac{\sqrt{S_T S_M}}{\rho_0 c}\sec\theta\sin(kl\cos\theta) \tag{9.77}$$

$$a_{22} = \sqrt{\frac{S_T}{S_M}}(\cos(kl\cos\theta) + \tan\theta\sin(kl\cos\theta)) \tag{9.78}$$

where S_T is the area of the throat, $S_M = S_T e^{ml}$ is the area of the mouth, l is the length of the horn from the throat to the mouth, and $\theta = \arcsin(m/2k)$. The throat impedance of a finite exponential horn is plotted in Fig. 9.10.

Finite hyperbolic horn

The matrix elements are given by

$$a_{11} = \sqrt{\frac{S_M}{S_T}}(\cos(kl\cos\theta) - \beta\tan\theta\sin(kl\cos\theta)) \tag{9.79}$$

$$a_{12} = j\frac{\rho_0 c}{\sqrt{S_T S_M}}\sec\theta\sin(kl\cos\theta) \tag{9.80}$$

$$a_{21} = j\frac{\sqrt{S_T S_M}}{\rho_0 c}\left((\beta - \alpha)\sin\theta\cos(kl\cos\theta) + \left(1 + (\alpha\beta - 1)\sin^2\theta\right)\sec\theta\sin(kl\cos\theta)\right) \tag{9.81}$$

$$a_{22} = \sqrt{\frac{S_T}{S_M}}(\cos(kl\cos\theta) + \alpha\tan\theta\sin(kl\cos\theta)) \tag{9.82}$$

where S_T is the area of the throat,

$$S_M = S_T(\cosh(l/x_T) + \alpha\sinh(l/x_T))^2$$

is the area of the mouth, l is the length of the horn from the throat to the mouth, and $\theta = \arcsin(1/kx_T)$. The quantity β is given by

$$\beta = \sqrt{\frac{S_T}{S_M}}(\sinh(l/x_T) + \alpha\cosh(l/x_T)). \tag{9.83}$$

The throat impedance of a finite hyperbolic horn is plotted in Fig. 9.10.

Truncation effects

Whenever the bell diameter is not large or when the horn length is short, it is not possible to use the infinite approximation for the throat impedance. Instead we must use the exact equation of Eq. (9.64). However, we see from Fig. 9.10 that, for a given size horn, the parabolic and conical horns are closer to the infinite ideal of Fig. 9.9 than are the exponential and hyperbolic types. To illustrate what the words "large bell diameter" and "long length" mean, let us refer to Fig. 9.11 for a finite exponential horn of various sizes.

If the circumference of the mouth of the horn divided by the wavelength is less than about 0.5 (i.e., the diameter of the mouth divided by the wavelength is less than about 0.16), the horn will resonate like a cylindrical tube, i.e., at multiples of that frequency where the length is equal to a half wavelength. This condition is shown clearly by the two lower-frequency resonances in Fig. 9.11a.

When the circumference of the mouth of the horn divided by the wavelength is greater than about 3 (i.e., diameter divided by wavelength greater than about 1.0), the horn acts nearly like an infinite horn. This is shown clearly by comparison of c and d of Fig. 9.11, for the region where f/f_c is greater than about 2, which is the case where the ratio of mouth diameter to wavelength exceeds 0.5.

In the frequency region where the circumference of the mouth to wavelength ratio lies between about 1 and 3, the exact equation for a finite exponential horn (Eq. (3.49)) must be used, or the results may be estimated from a and b of Fig. 9.11.

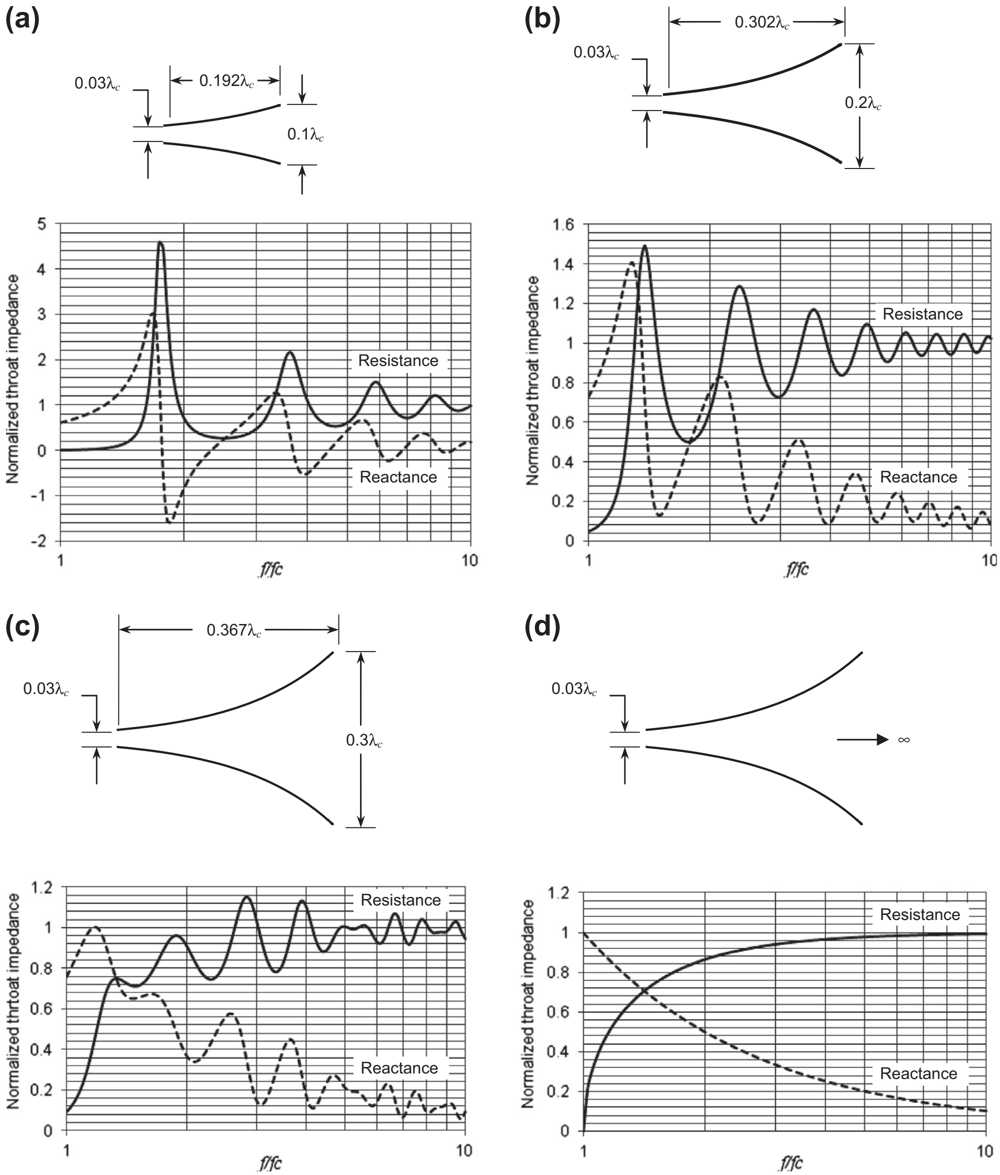

Figure 9.11 Graphs showing the variation in specific acoustic impedance at the throat of four exponential horns as a function of frequency with bell diameter as the parameter. The cutoff frequency $f_c = mc/4\pi$ and the throat diameter $= 0.03\ c/f_c$; both are held constant. Bell circumferences are (a) $C = 0.314\lambda_c$, (b) $C = 0.628\lambda_c$, (c) $C = 0.942\lambda_c$, and (d) $C = \infty$. The mouth of the horn is assumed to be terminated in an infinite baffle.

When the length of the horn becomes less than one-quarter wavelength, it may be treated as a simple discontinuity of area such as was discussed in Section 4.8 (pp. 131 to 133).

Obviously, if one chooses a certain mouth area and a throat area to obtain maximum efficiency, the length of the horn is automatically set by the flare constant m, which is in turn directly dependent on the desired cutoff frequency.

Nonlinear distortion

A sound wave produces an expansion and a compression of the air in which it is traveling. We find from Eq. (2.6) that the relation between the pressure and the volume of a small "box" of the air at 20°C through which a sound wave is passing is

$$P = \frac{0.726}{V^{1.4}} \tag{9.84}$$

where

V is specific volume of air in $\mathrm{m^3/kg} = 1/\rho_0$

P is absolute pressure in bars, where 1 bar $= 10^5$ Pa

This equation is plotted as curve AB in Fig. 9.12.

Assuming that the displacement of the diaphragm of the drive unit is sinusoidal, it acts to change the volume of air near it sinusoidally. For large changes in volume, the pressure built up in the throat of the horn is no longer sinusoidal, as can be seen from Fig. 9.12. The pressure wave so generated travels away from the throat toward the mouth.

If the horn were simply a long cylindrical pipe, the distortion would increase the distance the wave progressed according to the formula (air assumed) [14,15]

$$\frac{p_2}{p_1} = \frac{\gamma + 1}{2\sqrt{2}\gamma} k \frac{p_1}{P_0} x = 1.21 k \frac{p_1}{P_0} x \tag{9.85}$$

where

p_1 is rms sound pressure of the fundamental frequency in Pa.

p_2 is rms sound pressure of the second harmonic in Pa.

P_0 is atmospheric pressure in Pa.

$k = \omega/c = 2\pi/\lambda$ is wave number in $\mathrm{m^{-1}}$.

$\gamma = 1.4$ for air.

x is distance the wave has traveled along the cylindrical tube in m.

Eq. (9.85) breaks down when the second-harmonic distortion becomes large, and a more complicated expression, not given here, must be used.

In the case of an exponential horn, the amplitude of the fundamental decreases as the wave travels away from the throat, so that the second-harmonic distortion does not increase linearly with distance. Near the throat it increases about that given by Eq. (9.85),

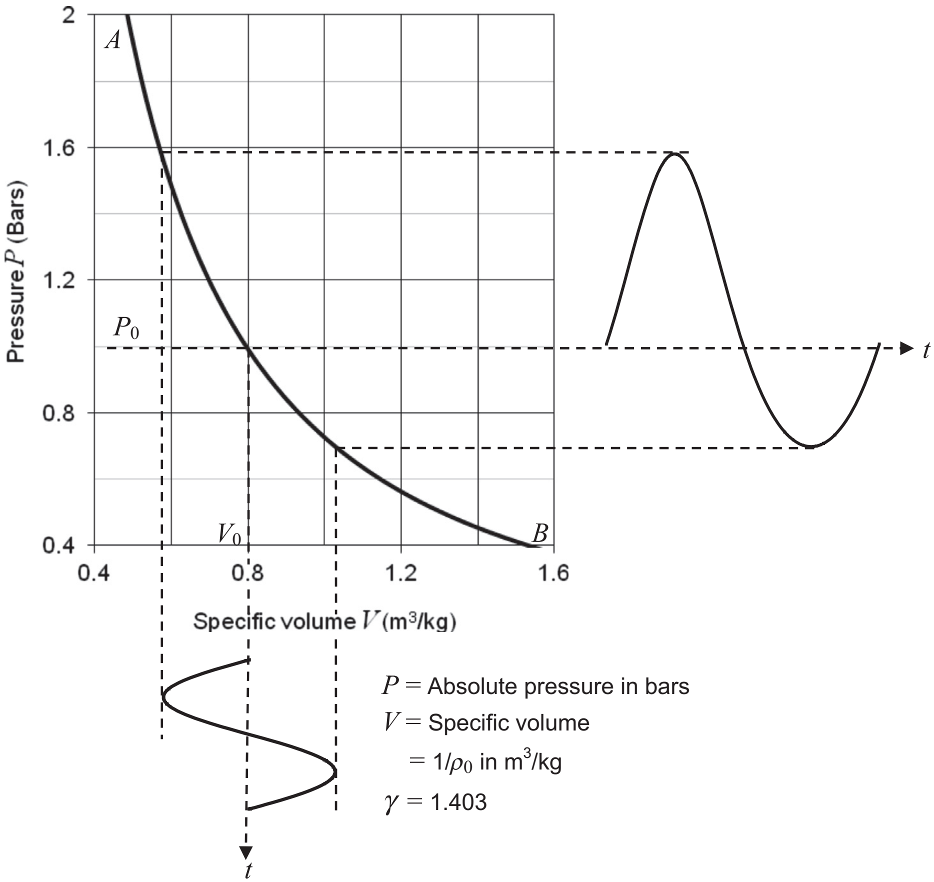

Figure 9.12 Plot of the gas equation $PV\gamma = 1.26 \times 10^4$, valid at 20°C. Normal atmospheric pressure (0.76 m Hg) is shown as $P_0 = 1$ bar.

but near the mouth the pressure amplitude of the fundamental is usually so low that very little additional distortion occurs.

The distortion introduced into a sound wave after it has traveled a distance x down an exponential horn for the case of a constant power supplied to unit area of the throat is found as follows:

1. Differentiate both sides of Eq. (9.85) with respect to x, so as to obtain the rate of change in p_2 with x for a constant p_1. Call this Eq. (9.85a).
2. In Eq. (9.85a), substitute for p_1 the pressure $p_T e^{-mx/2}$, where p_T is the rms pressure of the fundamental at the throat of the horn in Pa and m is the flare constant.
3. Then let $p_T = \sqrt{I_T \rho_0 c}$, where I_T is the intensity of the sound at the throat in W/m^2 and $\rho_0 c$ is the characteristic acoustic impedance of air in rayls.
4. Integrate both sides of the resulting equation with respect to x.

This yields:

$$\text{percent second} - \text{harmonic distortion} = \frac{50(\gamma+1)}{\gamma P_0}\sqrt{\frac{I_T \rho_0 c}{2}}\frac{f}{f_c}\left(1-e^{-mx/2}\right). \quad (9.86)$$

For an infinitely long exponential horn, at normal atmospheric pressure and temperature, the equation for the total distortion introduced into a wave that starts off sinusoidally at the throat is

$$\text{Percent second} - \text{harmonic distortion} = 1.22\frac{f}{f_c}\sqrt{I_T} \times 10^{-2} \quad (9.87)$$

where

f is driving frequency in Hz.

f_c is cutoff frequency in Hz.

I_T is intensity in W/m^2 at the throat of the horn.

Eq. (9.87) is shown plotted in Fig. 9.13. Actually, this equation is nearly correct for finite horns because most of the distortion occurs near the throat.

Eq. (9.87) reveals that, for minimum distortion, the cutoff frequency f_c should be as large as possible, which in turn means as large a flare constant as possible. In other words, the horn should flare out rapidly to reduce the intensity rapidly as one travels along the horn toward the mouth.

Unfortunately, a high cutoff frequency is not a feasible solution for horns that are designed to operate over a wide frequency range. In this case, it is necessary to operate the horn at low power at the higher frequencies if the distortion is to be low at these

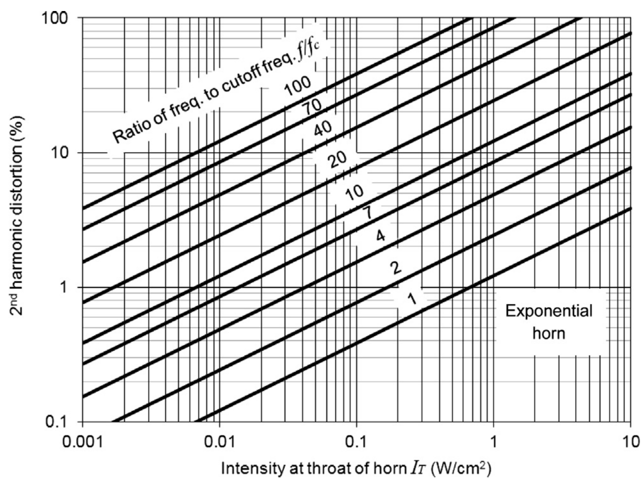

Figure 9.13 Percentage second-harmonic distortion in an exponential horn as a function of the intensity at the horn throat with the ratio of the frequency to the cutoff frequency as parameter.

frequencies. This goal is achieved automatically to some extent in reproducing speech and music because above 1000 Hz the intensity for these sounds decreases by about a factor of 10 for each doubling of frequency.